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LAMBERT'S TARGETING FOR CROSS-PRODUCT STEERING IN THE AS-503A LOI SIMULATION MANEUVER

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PROJECT APOLLO

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LAMBERT'S TARGETING FOR CROSS-PRODUCT STEERING IN THE AS-503A LOI SIMULATION MANEUVER

By Robert F. Wiley

SUMMARY

The question to be resolved in this study is whether or not cross-product steering using a Lambert's targeting scheme can be used for the AS-503A LOI simulation maneuver; and if so, how should the values of the targeting parameters be chosen? The LOI maneuver is to be performed near perigee of a high ellipse resulting from the AS-503A TLI simulation burn.

It was not certain at the initiation of the study that a practical targeting scheme would result. However the results of the study show that the proposed Lambert's scheme is practical, conceptually simple, relatively insensitive to dispersions, and can be implemented with currently planned Real-Time Computer Complex (RTCC) logic. The proposed targeting scheme achieves the target ellipse apogee and perigee altitudes and misses the target ellipse geographic position on the earth at the first perigee following the burn by less than 1.0° in latitude and less than 2.5° in longitude.

The results showed that the optimum targeting parameters should be as follows:

(1) <u>Target vector</u>--chosen at a true anomaly of 270° from the osculating conic at perigee of a precision ellipse having an apogee altitude of 200 n. mi. The precision ellipse is generated by an impulsive maneuver at perigee of the high ellipse.

(2) <u>Time of ignition</u>--equals time at perigee minus one half the time of burn.

(3) <u>Time of flight</u>--equals time at the target vector on the osculating conic associated with the target ellipse minus the time at engine ignition.

The ARRS program can be used in real time to generate and update these targeting parameters.

INTRODUCTION

In the early planning phases of the lunar mission, there was a different targeting scheme for each of the three major, nominal CSM-controlled maneuvers - TLI, LOI, and TEI. However, it was recently discovered that the spacecraft onboard computer would be short of storage space. Consequently, an effort was made to save some of this space by performing several of the maneuvers with only one targeting scheme. Thus a Lambert's targeting scheme was proposed by MIT to target cross-product steering for TLI and TEI[1] for the lunar mission.

The MIT cross-product steering law used in the CSM-controlled burns depends on defining a velocity-to-be-gained vector, which is the difference between a required velocity and the present CSM velocity. The different targeting schemes are merely means of calculating this required velocity, that is, the velocity the CSM must have at its present position and velocity to obtain the desired target conditions. Lambert's targeting computes conically the required velocity from the present CSM position vector, a target vector, and the time of flight between the two vectors. Reference 1 presents a more complete discussion of cross-product steering.

It is, of course, impossible for the CSM to attain the required velocity instantaneously; it must thrust for a finite length of time. During the burn, Lambert's problem is solved onboard every 4 seconds and the solution is extrapolated so that at each point through the powered-flight path, a new conic is computed to transfer to the target vector. Therefore, it is not immediately obvious how to select the initial transfer conic (i.e., the target parameter values) to insure that the conic at burnout will give the desired end conditions. (The conic at burnout will only pass through the target vector in the time of flight minus the burn time; it need not necessarily have the desired end conditions such as apogee altitude.) In the case of the AS-503A LOI maneuver, this is further complicated by a short time of flight (generally less than 1.2 hours). Thus, it was not certain at the outset that Lambert's steering for the AS-503A LOI burn could be targeted in a simple manner with no generalized iterations.

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[1]This Lambert's scheme may also be used for some of the return-to-earth abort maneuvers.

The AS-503A LOI simulation burn will consist of a transfer from a high ellipse to a low ellipse. The principal objective of the burn is to lower the apogee altitude of this high ellipse from 3950 to 200 n. mi. and perform a CSM burn that is at least as long as the LOI burn for the lunar mission.

The transfer to the low ellipse will occur at perigee of the high ellipse. At the time of writing, the perigee altitude of the high ellipse should be approximately 150 n. mi. which is high enough to insure tracking but low enough to allow an RCS deorbit. This altitude of perigee will be obtained by raising the altitude of perigee (by a simulated midcourse correction at second apogee) of the ellipse resulting from the TLI simulation. This ellipse (with the raised perigee) will be called the high ellipse, that is, the ellipse on which the LOI simulation burn is made. There is no restriction on the line of apsides or the groundtrack; once the nominal LOI burn is established, the LM maneuvers may be developed to satisfy mission tracking requirements.

This study was performed with the Apollo Reference Mission Program (ARMP1). The effects of ullage, thrust buildup, and tailoff were not considered.

An empirical, qualitative prediction of the behavior of Lambert's targeting in cross-product steering will be published in a memorandum which will summarize the results of this study as well as one on a spacecraft-guided AS-503A TLI simulation.

SYMBOLS

ARMP	Apollo Reference Mission Program
ARRS	Apollo RTACF Rendezvous System
CSM	command and service modules
GPMP	General Purpose Maneuver Processor
h_a	altitude of apogee, n. mi.
h_p	altitude of perigee, n. mi.

λ	longitude, degrees
LOI	lunar orbit insertion
ω	argument of perigee, degrees
ϕ	latitude, degrees
RCS	reaction control system
R_T	target vector on the osculating conic
RTACF	Real-Time Auxiliary Computing Facility
t_{bu}	backup time before perigee to start the burn, hours. $t_{bu} = .064$ means that the burn is started .064 hours before arriving at perigee
t_{burn}	burn time, hours
t_c	coast time on the osculating conic from perigee to the target vector, hours
TEI	transearth injection
t_f	time of flight from burn initiation to the target vector, may be greater or smaller than $t_{f\ imp}$, hours
$t_{f\ imp}$	impulsive time of flight, coast time on the osculating conic and the backup time on the high ellipse to start the burn, $t_{f\ imp} = t_c + t_{bu}$, hours
t_{ign}	time of engine ignition, hours
TLI	translunar injection

TARGETING PARAMETERS

The targeting parameters are those parameters which may be changed in order to obtain the desired end conditions subject to mission constraints.

(1) Target vector, R_T.- This vector is selected from a two-body ellipse (the target conic) defined to be the osculating conic at perigee of the desired precision orbit. Both the parameters of the conic and the true anomaly at which the target vector is selected may be varied. However, only the true anomaly of the target vector was considered a targeting parameter for this study.

(2) Time of ignition, t_{ign}.- Changes in the time of ignition are equivalent to changes in the CSM initial position.

(3) Time of flight, t_f.- This is the last of three inputs to Lambert's problem. This may be $t_{f\ imp}$ or some value greater or smaller.

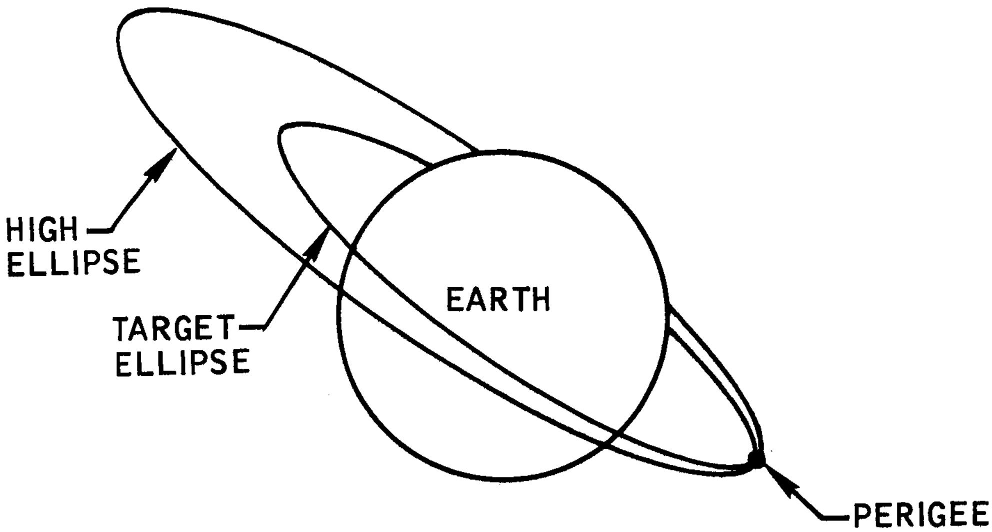
(4) The cross-product steering constant, c.- This constant controls the direction and rate of change of direction of the thrust vector. Since in Lambert's problem, the CSM is targeted only for a time of arrival at the target vector, c can control on which conic orbit of a family of possible orbits passing through the target vector the CSM will burn out.

The target ellipse is a precision orbit generated impulsively at perigee and defined by a 200-n. mi. altitude of apogee and an altitude of perigee equal to the altitude of perigee of the high ellipse (i.e., the ellipse that resulted from raising perigee of the TLI simulation ellipse). The argument of perigee is also that of the high ellipse. Apogee and perigee altitudes and the argument of perigee were measured at the first perigee after the LOI burn. The altitude of apogee is the two-body radius of apogee minus the radius of the earth, while the altitude of perigee is the actual height above the Fischer ellipsoid.

The target conic was generated by taking the state vector at second perigee of the high ellipse, changing the velocity and azimuth angle[2],

[2]Changing the azimuth angle causes the target conic plane to be inclined to the high ellipse plane. This insures that the CSM and the spent S-IVB will not recontact each other.

and coasting two-body. Note in the sketch below that the target conic intersects the high ellipse at perigee.



When this study was initiated the ellipse resulting from the TLI simulation burn measured 3950 by 109 n. mi. in altitude. Perigee was not to be raised because the RCS budget at that time would not allow an RCS deorbit from a higher perigee. There was also to be no plane change at perigee. Therefore, the target conic for this study measured 206 by 109 n. mi. and was in the same plane as the high ellipse. After the targeting scheme was developed using this target conic, it was checked by using perigee altitudes up to 150 n. mi. and by making a 1° plane change at perigee.

PRELIMINARY CHOICE OF TARGET

Three target vectors were chosen for investigation to get an idea of an optimum place to select the target vector for the target conic used in this study.

Target vectors	True anomaly	Time of coast
R_T I	91.3°	0.375 hours
R_T II	267.5°	1.125 hours
R_T III	297.6°	1.250 hours

The time of ignition (t_{ign}) or the back-up time from perigee (t_{bu}) to start the burn was arbitrarily chosen to be .087 hours or 65 to 70 percent of the near-nominal burn time (t_{burn}). The cross-product steering constant (c) was set at 1.0, because it had previously been found that this value of c generally gives a minimum ΔV burn. The time of flight was chosen as a variable because it seemed to be the most sensitive variable. For each target various times of flight were chosen, and the results compared.

Figures 1 through 4 present plots of apogee altitude (h_a), perigee altitude (h_p), argument of perigee (ω), and ΔV, respectively, versus $t_f - t_c$. The quantity $t_f - t_c$ was chosen as the common abscissa because the conic coast time, t_c, depends on the target true anomaly, giving different total times of flight for each target.

Figure 1, apogee altitude versus $t_f - t_c$, shows that R_T I is much more sensitive than either R_T II or III; that is, there is a much greater variation in apogee altitude for a given time for R_T I than for R_T II or III. The sensitivity of R_T I is probably due to the short t_c associated with it. (Remember that this short t_c was expected to be a problem.) Note that for R_T II the target apogee is obtained at $t_f - t_c = t_{bu} =$ .087 hours, or $t_f = t_{bu} + t_c$; that is, $t_f = t_{f\ imp}$.

The sensitivity of R_T I is again shown in figures 2 and 3. By comparing figure 3 to figures 1 and 2, it can be seen that the $t_f - t_c$ value at which h_a and h_p become stable on the R_T I curve is about where the argument of perigee becomes most sensitive. Note that R_T II is about as sensitive as R_T III on all three figures.

Figure 4 is a plot of the ΔV required for the LOI burn as a function of $t_f - t_c$. To lower apogee altitude from 3950 to about 200 n. mi. using R_T I requires less ΔV than using R_T II or III. For example, to lower apogee to 310 n. mi. requires 3912 fps ΔV with R_T I as compared to 3948 fps ΔV with R_T II. At around 200-n. mi. apogee altitudes, the ΔV's are equal, and below 200 n. mi., R_T II and III are cheaper than R_T I. For example, to lower apogee to 163 n. mi. requires 100 fps more ΔV with R_T I than R_T II. It is suspected that the dividing point at 200 n. mi. arises because that is the apogee altitude of the target conic. This could be verified by redoing this preliminary target choice study using different apogee altitudes for the target conics.

Therefore, since there is little difference in ΔV to obtain the desired apogee altitude and R_T I is so sensitive, R_T II will be used for the target vector in the study. R_T II was chosen over R_T III (which has a slightly longer t_c) because the target apogee was obtained at $t_f = t_{f\ imp}$ and because real-time updating logic that selects a target vector at 270° is planned. This logic is the GPMP portion of the ARRS program, commonly called the "Apollo Monster." The choice of R_T II was checked after the targeting scheme was developed and will be discussed in a later section.

DETERMINATION OF THE TARGETING SCHEME

The principal objective of the AS-503A LOI burn is to lower the altitude of apogee of the high ellipse to 200 n. mi. There is no strict requirement on the resultant perigee altitude. However, since this burn will occur near perigee, the higher the perigee altitude can be kept, the better will be the tracking coverage[3]. Thus, the targeting scheme

[3]If the altitude (a) drops 11 n. mi. at a point during the burn, the radius of the tracking circle decreases: $R_{new} = R_{old} \dfrac{a_{old} - 11}{a_{old}}$. For our case this is about .90 R_{old} or the radius is only .90 of what it was 11 n. mi. higher.

will attempt to keep the perigee altitude of the low ellipse equal to that of the high ellipse. Any ΔV penalties associated with this will be checked after the targeting scheme has been developed. The argument of perigee of the low ellipse need not be controlled, but should be predictable after the LOI burn.

Targeting With The Steering Constant, c, Equal to 1.00

Some previous thrusting simulations have shown that many times a value of c around 1.00 results in the minimum ΔV. Therefore, to initiate this study, c has been set equal to 1.00.

One-parameter targeting - time of ignition (t_{ign}).- Here, only one targeting parameter, t_{ign}, will be varied. The time of flight is $t_{f\ imp}$, c is 1.00, and the true anomaly of the target is 267.5° (from the preliminary choice of target).

Figures 5 through 8 present apogee altitude, perigee altitude, argument of perigee, and ΔV, respectively, as a function of t_{bu}. These graphs show that with the parameter values ($c = 1.00$, $t_{f\ imp}$), the burn having $t_{bu} = .0766$ hour results in an ellipse closest to the target ellipse. The parameters of the resulting ellipse and the target ellipse are given below for comparison.

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps	t_{bu}, hr	$\frac{t_{bu}}{t_{burn}}$, nd
Burn ellipse	98.6	208	131.8	4003.3	.0766	.596
Target conic	108.99	205.85	128.04	---	---	---

Note, however, that the correct h_a is obtained with $t_{bu} = .08$.

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps	t_{bu}, hr	$\frac{t_{bu}}{t_{burn}}$, nd
Burn ellipse	99.7	206	132.8	4010	.08	.622
Target conic	108.99	205.86	128.04	---	---	---

Obviously freedom to change more than one parameter is necessary to obtain the desired apogee and perigee altitudes.

Two-parameter targeting - time of ignition (t_{ign}) and time of flight (t_f).- Now t_f as well as t_{bu} (i.e., t_{ign}) will be varied. c will still be 1.00, and the target true anomaly will remain 267.5°. This will be termed t_{bu}-t_f targeting.

Table I shows that there are many combinations of t_{bu} and t_f that will give a certain altitude of apogee. Notice that for t_{bu} = .0766 hour, $t_f = t_{f\ imp}$ (i.e., $t_{bu} + t_c$). For $t_{bu} > .0766$, t_f to obtain the desired h_a is $> t_{f\ imp}$ and for $t_{bu} < .0766$, t_f to obtain the desired h_a is $< t_{f\ imp}$. Thus, the combination of t_{bu} and $t_{f\ imp}$ to give a desired h_a forms a boundary; i.e., a $t_{bu} > t_{bu}$ boundary requires a $t_f > t_{f\ imp}$ to obtain that h_a.

In a 200 x 100 n. mi. altitude ellipse, it takes 1.78-fps ΔV applied impulsively at apogee to raise perigee 1 n. mi. (this is good for Δh_p to at least 20 n. mi.). Thus, table II may be created by raising the perigee of table I to 108.99, the target value. This will allow a comparison of the different combinations of t_{bu} and t_f to perform the LOI burn. For this comparison, figure 9, which shows total change in velocity versus t_{bu}/t_{burn}, was created from table II. This graph shows that t_{bu} should be about 55 percent of t_{burn} for a minimum ΔV. (As a rough approximation, we can say t_{bu} should equal 1/2 t_{burn} or be slightly greater for minimum ΔV.)

By again referring to table I, figure 10, h_p versus t_{bu}, was created. Each point on the curve of figure 10 has an h_a = 208 n. mi. This shows that there should be two t_{bu}'s where h_p is 109 n. mi., when h_a is searched in to 208 n. mi. by varying t_f. That is, the desired h_a and h_p can both be obtained. One of these combinations gives:

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps	t_{bu}, hr	$\frac{t_{bu}}{t_{burn}}$, nd	t_f, hr	$t_f - t_c$, hr
Burn ellipse	109.34	206.79	123.77	4218.57	.031	.231	1.151	.020
Target conic	108.99	205.86	128.04	---	---	---	---	---

Here, t_{bu} is much smaller than .5 t_{burn}, giving a large ΔV[4].

One last conclusion may be drawn from this section. Varying one parameter (t_{ign}) allowed control of one of the end conditions; varying two parameters (t_{bu} and t_f) allowed control of two of the end conditions. Thus, it seems that for every end condition desired, control of that many target variables is necessary, and the ΔV will increase as the number of desired end conditions is increased. Thus, there is a one-for-one control of the end conditions.

Apparently there is only a small range of allowable values of t_{bu} around 0.5 t_{burn} which avoid excessive ΔV penalties. Thus to obtain the desired h_a and h_p and keep ΔV to a minimum, t_{bu} must stay in this range and a target parameter other than t_f must be changed. Thus

[4] A ΔV of 4200 fps is high relative to the ΔV's so far. Later we will see that the same end conditions can be met for a ΔV of 4026 fps. It is, perhaps, too early to say that minimum ΔV will result at t_{bu}/t_{burn} = .5. This approximation will be better substantiated in a later section.

in this next section c will not be constrained to equal 1.00 but will be a target variable[5].

Targeting With The Steering Constant, c, As A Target Parameter

A targeting scheme in which t_{bu} and c were varied to obtain the desired end conditions with $t_f = t_{f\ imp}$ was investigated. The true anomaly of the target vector was still 267.5°. This is called c-t_{bu} targeting. From the results of the previous section it was found that there is only a small range of back-up times in which the ΔV is not excessive. Given this small range of acceptable values, will this scheme work? To summarize before looking at the data: it was found that varying c and t_{bu} will give acceptable results; thus, this will be the proposed targeting scheme for the AS-503A simulation burn.

This part of the study was done by selecting various values of t_{bu} and trying different values of c for each of them. The burn results are shown in figures 11 through 14.

Figure 11 presents c versus t_{bu} with lines of constant h_a and h_p. This figure shows trends only: it is not accurate because the altitudes were rounded off to the nearest nautical mile and the number of points plotted was small. However, note that for a particular value of c, the same value of h_p is obtained at two different t_{bu}'s. This is a more general view of the results of the previous section, t_{bu}-t_f targeting with $c = 1.00$, which showed that one value of h_p was obtained at two different values of t_{bu}.

Figure 12 shows h_p as a function of t_{bu} with lines of constant c. This is the most important graph in the report since the things that show on it are the basis for the c-t_{bu} targeting scheme. First, note

[5]We could, of course, keep $c = 1.00$ and change the target vector true anomaly; this is done later in the study.

that in a region where t_{bu} is about 45 to 55 percent of t_{burn}, h_p is roughly independent of t_{bu}[6]. This means that in this region, once one fixes a value of c, the value of the perigee altitude is fixed to within 0.5 n. mi. Thus, with a good estimate of the burn time [e.g., from solving $\Delta V = g_o I_{sp} \ln (M_o/M_o - \dot{M}t)$ for t_{burn} with ΔV obtained from an impulsive maneuver] one can use $t_{bu} = 0.5\ t_{burn}$ and find a value of c that will guarantee a certain h_p ($\pm$ .5 n. mi.) when t_{bu} is between .45 and .55 t_{burn}. One can then vary t_{bu} to obtain the correct h_a. It also appears as if this may make the targeting more stable to dispersions than other schemes, such as the t_{bu}-t_f scheme of the last section. This will be checked in a later part of this study.

Figure 13 shows h_p versus c with lines of constant t_{bu}. First, note that h_p is a linear function of c. Second, note that the t_{bu} = .065 hours ($\simeq$ .50 t_{burn}) line forms a boundary of the accessible h_p and c combinations; that is, any combination of h_p and c to the left of the t_{bu} = .065 hours ($\simeq$ .50 t_{burn}) line cannot be reached using the target vector at 267.5° and $t_{f\ imp}$. This result can be generalized by saying that $t_{bu} = 0.5\ t_{burn}$ forms an inflection point on the constant c lines of the h_p versus t_{bu} graph, and this inflection point marks the t_{bu} of lowest h_p for a particular value of c. It appears reasonable to extend these empirical results and say that the shapes of figures 12 and 13 will hold for other target vectors although the scales will probably shift[7].

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[6]This is caused by the quadratic nature of the lines of constant c.

[7]A thorough study to substantiate this has not been done. However, a few limited cases show that this scale does shift; that is, if c = .50 gives a certain h_p for the target at 267.5°, it will give a different h_p for a different target vector true anomaly on the same target conic.

Figure 14 shows h_a as a function of t_{bu} with lines of constant c. First note that this is, in general, not a linear relation. Second, note that an increase in the backup time results in a decrease in the apogee altitude (which may also be seen in figure 11).

Thus given the characteristics of the spacecraft used in this study, the values of the targeting parameters may be computed. These are not intended for any real-time use, only as comparisons in the study. These are called the study nominal values. For a burn targeted with c = 0.49, t_{bu} = .064 hours, and $t_f = t_{f\ imp}$, the end conditions shown in the table below resulted.

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps	$\frac{t_{bu}}{t_{burn}}$, nd	ϕ, deg	λ, deg
Burn ellipse	109.0	206	126.4	4025.6	0.496	25.39	234.51
Target conic	109.0	206	128.04	---	---	24.56	236.75

Comparison of c-t_{bu} and t_{bu}-t_f Targeting

The table below clearly shows the superiority of c-t_{bu} over t_{bu}-t_f targeting. More than 1000 lb of fuel is saved by c-t_{bu} targeting and, although both methods obtain the target apogee and perigee c-t_{bu} targeting comes closer to achieving the target line of apsides and latitude and longitude at the first perigee after the burn.

	h_p, n. mi	h_a, n. mi.	ω, deg	ΔV, fps	Fuel used, lb	ϕ, deg	λ, deg
c-t_{bu}	109.01	205.77	126.38	4025.61	29507.36	25.39	234.51
t_{bu}-t_f	109.34	206.79	123.77	4218.57	30648.41	26.27	232.03
Target conic	108.99	205.86	128.04	---	---	24.56	236.75

Checks On The Optimality of c-t_{bu} Targeting

1. c-t_{bu} targeting compared to a two-burn maneuver with $c = 1.00$

A two-burn maneuver would require another burn after the LOI burn. This is undesirable because it will extend an already too long crew work day and, require more ground support and possibly new ground programs. However, if enough ΔV could be saved, this maneuver might be worth doing.

To do this comparison the target vector will remain at a true anomaly of 267.5°, c will be set equal to 1.00, and t_{bu} will be varied to obtain the correct h_a. The results are:

Case	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps	t_{bu}, hr	$\frac{t_{bu}}{t_{burn}}$, nd
1. $c = 1.00$	96.75	213.48	129.03	3988.18	.064	.500
2. $c = 1.00$	99.70	205.78	132.57	4014.42	.080	.621
3. $c = 0.49$ (study nominal)	109.01	205.77	126.38	4025.61	.064	.496
Target conic	108.99	205.86	128.04	---	---	---

In case 1, h_a could be lowered by a second burn at the first perigee after the LOI burn, which would cost about 13 fps. Thus 25 fps would be saved. This is not enough of a saving to make up for the extra work loads. Obtaining a 109-n. mi. h_p in addition to the 206 n. mi. h_a would require a third burn.

Case 2 already gives the correct h_a at a saving of 11 fps. The additional 11 fps for case 3 (the study nominal results) results in h_p 9 n. mi. higher than that of case 2 (giving better tracking coverage throughout the burn) and a t_{bu}/t_{burn} ratio of $\simeq$ .5. The importance of this ratio, t_{bu}/t_{burn}, will be discussed later under real-time considerations. (The ratio equal to .5 will allow us to use the GPMP in the ARRS program.) Doing a second burn to raise h_p would cost an additional 16 fps, giving no ΔV savings overall.

2. <u>c-t_{bu} targeting compared to targeting with $c = 1.00$, a different target true anomaly</u>.- Since moving away from $t_{bu}/t_{burn} = 0.5$ seems to be expensive, we can form another optimality check by keeping $t_{bu} = .064$ (= .5 t_{burn}) but setting $c = 1.00$ and changing the target vector true anomaly to obtain the correct h_a. The results of this are:

Target True Anomaly	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps	$\frac{t_{bu}}{t_{burn}}$, nd	c, nd
Target	108.99	205.86	128.04	---	---	---
267.5° (Study nominal)	109.01	205.77	126.38	4025.86	.496	.49
208.5°	101.41	206.78	122.72	4001.77	.492	1.00

It would take 15 fps to raise h_p of the 208.5°, $c = 1.00$ case to 109 n. mi. giving then only a savings of 9 fps, a savings not worth the extra burn. However, if we neglect h_p, 24 fps may be saved. This saving gives rise to three disadvantages not shared by the 267.5°, $c = .49$ (study nominal) case:

(1) The burn arc is about 34°, centered around the perigee point. This means that at burnout, the target and position vectors would be 208.5 - 17 = 191.5° apart. In the neighborhood of a 180° separation between target and position vectors, Lambert's routines do not work because the two vectors no longer uniquely determine a plane. Thus, it is undesirable for the target and position vectors to be near 180° apart[8].

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[8]MIT has modified the onboard Lambert's routine to handle the 180° case by defining the plane as the one determined by the present position and velocity vectors. However, I do not know what the effect of this modification will be on plane changes.

(2) h_p is lowered throughout the burn, limiting tracking coverage.

(3) A target vector of 209° true anomaly cannot be used in the GPMP[9].

3. Final check on the optimality of the target vector position.- As a final check on the overall optimality of $c-t_{bu}$ targeting with the target vector at 267.5°, two other target vector positions, 297.6° and 328.0° were chosen. $c-t_{bu}$ targeting was used to obtain h_a and h_p. The results are:

	Target conic	Target 267.5	Target 297.6	Target 328.0
h_p, n. mi.	108.99	109.01	108.97	108.64
h_a, n. mi.	205.86	205.77	204.54	206.26
ω, deg	128.04	126.38	127.39	128.44
ΔV, fps	---	4025.61	4026.34	4052.72
c, nd	---	0.49	0.43	0.30
t_{bu}, hr	---	0.064	0.0675	0.0885
t_{bu}/t_{burn}, nd	---	0.496	0.523	0.682
ϕ, deg	24.56	25.39	25.04	24.66
λ, deg	236.75	234.51	235.49	236.44

[9]There is one possible disadvantage that has not been checked out. The 208.5° case has a shorter t_c than the 267.5° case by 0.250 hour or about 21 percent of the 267.5° time of flight. Remembering how unstable the 90° case was, I feel that maybe the 208.5° case will not be as stable to dispersions as the 267.5° case.

It is possible to obtain all three parameters of the low ellipse. A target vector position at 328° gives the correct line of apsides and also the correct latitude and longitude at the first perigee after the burn indicating the CSM is at the right position on the low ellipse at the right time (it might not have ended up at the right point at the right time even if the inertial position of the line of apsides was correct)[10]. However, if t_{bu} and c are changed, the argument of perigee will not stay constant. (Remember, that for a particular target vector, one particular value of c would correspond to one particular value of h_p[11].) To show this the target was taken at 297.6° with $t_{bu} = .064$ and $c = .49$ (the study nominal values). This gives:

$h_p = 107.19$ n. mi. $h_a = 207.34$ n. mi. $\omega = 128.13°$.

Now when t_{bu} is changed to 0.0675 and c to 0.43 to obtain the correct h_a and h_p for this target position, a 0.74° change in the argument of perigee results:

$h_p = 108.97$ n. mi. $h_a = 204.54$ n. mi. $\omega = 127.39°$

These results lead to figure 15, which shows the argument of perigee versus target vector true anomaly. The argument of perigee was measured after the correct value h_a and h_p were obtained. Notice that, to the nearest .05°, this is a linear relation. Therefore, to obtain the correct argument of perigee after h_a and h_p are found, merely connect two known points with a straight line and find the target true anomaly at the desired argument of perigee. Then, when one has found the correct h_a and h_p for this target true anomaly, one will also have the correct argument of perigee.

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[10]There is another way that the argument of perigee might be controlled. Note that the difference between the target ω and the ω obtained with the target vector at 267.5° is 1.66°. Thus, the target conic could be generated at 1.66° past perigee and t_{bu} measured from there. Then, after the burn, the argument of perigee would be at 128.04°. This would also keep $t_{bu} = 1/2\ t_{burn}$, thereby maybe giving a lesser ΔV than that obtained by changing the target true anomaly. However, this might not put us on the target (generated at perigee) ground track. No attempt has been made to check this out, however.

[11]This held for $.45\ t_{burn} \leq t_{bu} \leq .55\ t_{burn}$.

These results using target true anomalies of 267.5°, 297.6° and 328.0° also support three conclusions drawn earlier:

(1) Minimum ΔV occurs with the t_{bu}/t_{burn} ratio around 0.5. With a target at 328.0°, $t_{bu}/t_{burn} = 0.682$ giving a ΔV of 27.11 fps greater than the ΔV for the target at 267.5° and $t_{bu}/t_{burn} = 0.496$[12].

(2) There is a one-for-one control of the end conditions. Three target parameters must be varied in order to obtain three end conditions.

(3) The more end conditions one wishes to control, the more expensive in ΔV it will be. This is because you are forced to move from optimal positions in order to control extra end conditions. Obviously, (1) and (3) are interdependent.

For the purposes of AS-503A, it is not necessary to obtain the target argument of perigee. Control of all three target parameters is outweighed by the ΔV penalty and the real-time considerations.

Real-Time Considerations

These considerations have to do with the real-time update of the targeting, caused by dispersions of the nominal trajectory. It is desirable to do as little real-time programing as possible; that is, let the GPMP do this in real time. Given a point on the orbit, the GPMP will perform an impulsive maneuver there and generate a two-body target ellipse. A target vector at 270° is then selected from this target ellipse. An impulsive ΔV is calculated, and the equation

$$\Delta V = g_o \; Isp \; \ln\left[M_o / \left(M_o - \dot{M} \; t \right) \right]$$

is solved for t_{burn}. The ignition time is then merely the time at the impulsive point minus 1/2 t_{burn}. We can see that the $c\text{-}t_{bu}$ targeting scheme with the target at 270° best fits into this program - both because the target is at 270° and because the t_{bu}/t_{burn} ratio is $0.496 \approx 0.50$.

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[12] One could argue against this by saying that it is the target position, not the t_{bu}/t_{burn} ratio that causes the increased ΔV. However, we saw before that for a particular target position, the ratio of .5 gave minimum ΔV. These results with different target positions thus do not disprove this conclusion.

Thus, the GPMP logic should be able to target large CSM burns in earth orbit. It is not anticipated that this would work for the lunar mission major CSM burns since they must be much more precise than those for AS-503A.

Thus the targeting scheme for the AS-503A LOI burn is recommended to be as follows:

Target vector: Selected from a two-body target ellipse at a true anomaly of 270°.

Time of ignition: Time at perigee of high ellipse minus 1/2 t_{burn}.

Time of burn: Computed from the ideal rocket equation:

$$\Delta V = g_o \text{ Isp} \ln \left[M_o / \left(M_o - \dot{M} t \right) \right]$$

Time of flight: Time at target minus t_{ign}

Two-body target ellipse: Generated by the GPMP to give the desired precision orbit.

Steering constant, c: Selected so that h_p of the high ellipse is the same as h_p of the low ellipse. This will be around 0.5.

The accuracy of this scheme will be further checked by Rendezvous Analysis Branch (RAB) and Mission Analysis Branch (MAB) going through this procedure. Results of the study in this internal note indicate the scheme will be acceptable.

Consideration of The Plane Change And Different High Ellipse Perigee Altitudes

The preceding AS-503A LOI targeting was done with a coplanar burn on a high ellipse having a perigee altitude of 109 n. mi. The necessary plane change during the burn and the different perigee altitudes that result from the ever changing RCS fuel budget available for deorbit must now be considered.

For each new perigee altitude a new target conic was generated with 1.0° plane change. A target vector at roughly 270° was selected. The LOI burn was simulated for each new h_p using the values of t_{bu} and c derived from the 109-n. mi. h_p, coplanar case. The time of flight was $t_c + t_{bu}$. The results are given in table III. They show:

(1) A 1.0° plane change requires no change of the coplanar targeting values. This means that if during the mission a different plane change must be made (say to give better tracking at a later time), a change of up to at least 1.0° should require only the generation of a new target conic and a corresponding new time of flight; c and t_{bu} could remain the same.

(2) The differences between the target and actual perigee altitudes for each different h_p case were smaller than 0.1 n. mi., and the differences between the target and actual apogee altitudes were smaller than 0.6 n. mi. This means that if nominal plans call for a h_p of 150 n. mi. and, due to excessive RCS usage, h_p can only be raised to 130 n. mi., the premission c and t_{bu} values would require no update.

(3) For each case, the t_{bu}/t_{burn} ratio was 0.49 to 0.50. Thus, the cases were good approximations of the proposed real-time targeting scheme.

Limitations and Advantages of This Targeting Scheme When Updating in Real Time

(1) There will be no control over the argument of perigee of the low ellipse; its location can be roughly predicted but not forced to a certain value. (It will be within 5° of the argument of perigee of the target ellipse.) This means also that a return to a nominal groundtrack cannot be absolutely guaranteed. Any maneuver to control the argument of perigee of the low ellipse must be done at the simulated midcourse correction on the high ellipse.

(2) Perigee altitude cannot be forced to a specific value. It could be changed about 10 n. mi. by updating c but there is no provision

for a c update in real time.[13] The value for c will guarantee that h_p high ellipse = h_p low ellipse to within about .5 n. mi.

(3) There will be control over the apogee altitude. With the target vector at 270° and a $t_{bu} = 1/2\ t_{burn}$, the apogee of the target ellipse can be obtained.

(4) There will be no additional or new RTCC logic necessary; no iterations will be necessary to update the target parameter values. These are perhaps the most important points of this scheme.

Brief Summary of This Section

We have seen here that the primary end condition, an apogee altitude of 200 n. mi., may be obtained in a number of ways without trying to control the other orbital parameters (perigee and argument of perigee). However, one method (target vector at 270°, $t_{bu} = 1/2\ t_{burn}$, c = .50) will also give the target perigee altitude for a little more ΔV, and would allow the use of the GPMP portion of the ARRS program to update the targeting parameters.

EVALUATION OF THE EFFECT OF DISPERSIONS

The stability of the targeting scheme recommended for use in the AS-503A LOI burn is examined in this section. Dispersions in thrust and ignition point as well as errors in the position measurement of the spacecraft are considered. This section presents an idea of how dispersions will affect this targeting scheme; this is not meant tc be a thorough dispersion analysis.

Thrust Dispersions

Using the target conic used to develop the scheme - 109 n. mi. perigee altitude and a coplanar burn - both the stability of $c\text{-}t_{bu}$ (the study

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[13] Any c update would have to be manually entered into the CMC by the crew

nominal values) and $t_{bu}-t_f$ targeting were checked. Dispersions resulting from 10%-low thrust, are shown in the following table:

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps
$c-t_{bu}$ (study nominal)	1.04	2	-2.64	21.96
$t_{bu}-t_f$	3.64	2	-4.39	112.68

This shows the superiority of $c-t_{bu}$ targeting. The targeting scheme stability to thrust dispersions appears to be acceptable. Note that in figure 17, h_p changes less than 0.5 n. mi. between -5 and +15 percent thrust for $c-t_{bu}$ targeting. Thus the suspicion that the quadratic nature of the h_p versus t_{bu} graph (fig. 12) would stablize h_p is confirmed.

Figures 16 through 19 show ΔV, h_p, h_a, and ω for the thrust variations considered.

Ignition Point Dispersions

This section still uses the coplanar, 190-n. mi. h_p target conic. The nominal targeting values have been used but ignition has occurred at times different from the nominal. Only the $c-t_{bu}$ targeting has been considered here. For engine ignition occuring 10.8 seconds late, the following dispersions result:

h_p, n. mi.	- .06
h_a, n. mi.	+9.0
ω, deg	-5.07
ΔV, fps	-14.51

Again, the targeting scheme's stability seems acceptable. Figures 20 through 23 present ignition point dispersions for ΔV, h_p, h_a, and argument of perigee, respectively. Again note the stability of h_p in figure 21.

Spacecraft Orbit Measurement Errors

This portion of the study was done with a different high ellipse than that used throughout this study. This ellipse has a perigee altitude of 117 n. mi. and a 1.0° plane change. Only $c\text{-}t_{bu}$ targeting was considered. The values of the targeting parameters are the ones for the nominal high ellipse. However, the radius of the high ellipse has been changed +2.5 n. mi. and -3.0 n. mi. in increments at a true anomaly of 214°. These new orbits have been propogated to the nominal ignition time and the maneuver performed. The results are shown in table IV. The maximum errors shown below seem acceptable.

h_p, n. mi.	3.0
h_a, n. mi.	4.0
ω, deg	0.4
ΔV, fps	10.

No velocity error measurement analysis has been done in this study.

TABLE I.- COMBINATIONS OF BACK-UP TIME (t_{bu}) AND TIME OF FLIGHT (t_f) TO OBTAIN DESIRED APOGEE ALTITUDE

[Steering constant, c, = 1.00]

t_{bu}, hours	t_f, hours	$\frac{t_{bu}}{t_{burn}}$, nd	$t_f - t_c$, hours	h_a, n. mi.	h_p, n. mi.	ω, deg	ΔV, fps
.0866	1.2144	.610	.0894	208	103.29	129.23	4035.07
.0766	1.2016	.596	.0766	208	98.62	131.83	4003.28
.0695	1.1930	.542	.0680	208	96.86	132.54	3996.49
.0666	1.1897	.519	.0647	208	96.51	132.40	3996.99
.0466	1.1675	.358	.0425	208	99.65	129.65	4071.82

TABLE II.- RESULTS OF IMPULSIVELY RAISING PERIGEES

GIVEN IN TABLE I AT APOGEE OF THE LOW ELLIPSE

[h_a = 208 n. mi., h_p = 109 n. mi.]

t_{bu}, hours	$\frac{t_{bu}}{t_{burn}}$, nd	h_p, n. mi.	ΔV, fps	Δh_p, n. mi.	ΔV to raise h_p, fps	Total ΔV, fps
.0866	.670	103.29	4035.07	5.70	10.15	4045.22
.0766	.596	98.62	4003.28	10.37	18.46	4021.74
.0695	.542	96.86	3996.49	12.13	21.59	4018.08
.0666	.519	96.51	3996.99	12.48	22.21	4019.20
.0466	.358	99.65	4071.82	12.34	21.97	4084.16

TABLE III.- TARGETING OF THE AS-503A LOI MANEUVER WITH THE PROPOSED TARGETING SCHEME

[Target parameter values: t_{bu} = 0.064 hr; c = 0.47; target true anomaly = 270°; $t_f = t_{fimp}$]

	Plane change when generating target conic, deg	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps	t_{bu}/t_{burn}, nd
(a) Targeting from high ellipse with 108.99-n. mi. perigee altitude						
Target	0.0	108.99	205.86	128.04	--	--
Burn results	---	109.01	205.77	126.34	4025.61	0.496
(b) Targeting from high ellipse with 116.84-n. mi. perigee altitude						
Target	1.0	116.84	210.14	124.27	--	--
Burn results	---	116.76	210.61	122.48	4046.60	0.494
(c) Targeting from high ellipse with 116.84-n. mi. perigee altitude						
Target	1.0	116.84	198.63	124.27	--	--
Burn results	---	116.86	199.14	122.02	4067.72	0.492
(d) Targeting from high ellipse with 149.84-n. mi. perigee altitude						
Target	1.0	149.84	202.56	124.26	--	--
Burn results	---	149.94	203.16	120.03	4055.20	0.488

TABLE IV.- RESULTS OF USING THE NOMINAL TARGET PARAMETER VALUES WITH DISPERSED ORBITS

Radius at true anomaly of 214°, n. mi.	ΔV, fps	h_p, n. mi.	h_a, n. mi.	ω, deg
Target	----	116.84	198.6	124.27
6770.3403	4076.15	119.94	195.2	122.43
6769.8403	4074.46	119.33	196.0	122.34
6769.3403	4072.76	118.71	196.8	122.25
6768.8403	4071.10	118.09	197.6	122.17
6768.3403	4069.41	117.48	198.3	122.09
6767.8403 (nominal)	4067.72	116.86	199.1	122.02
6767.3403	4066.03	166.24	199.9	121.95
6766.8403	4064.34	115.62	200.7	121.88
6766.3403	4062.66	115.00	201.5	121.81
6765.8403	4060.98	114.39	202.3	121.75
6765.3403	4059.30	113.77	203.1	121.68
6764.8403	4057.61	113.15	203.9	121.62

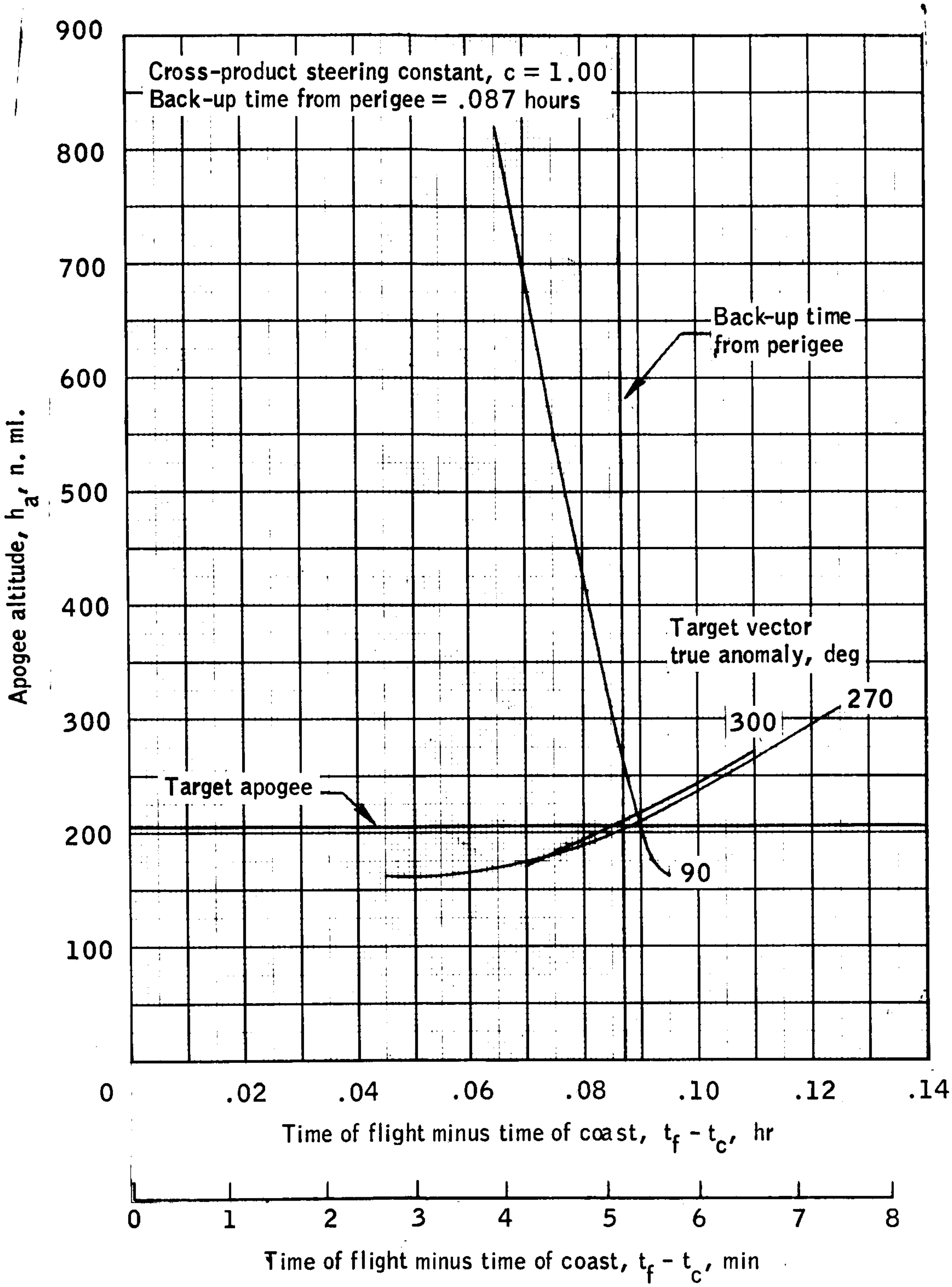

Figure 1.- Apogee altitude versus time of flight minus time of coast.

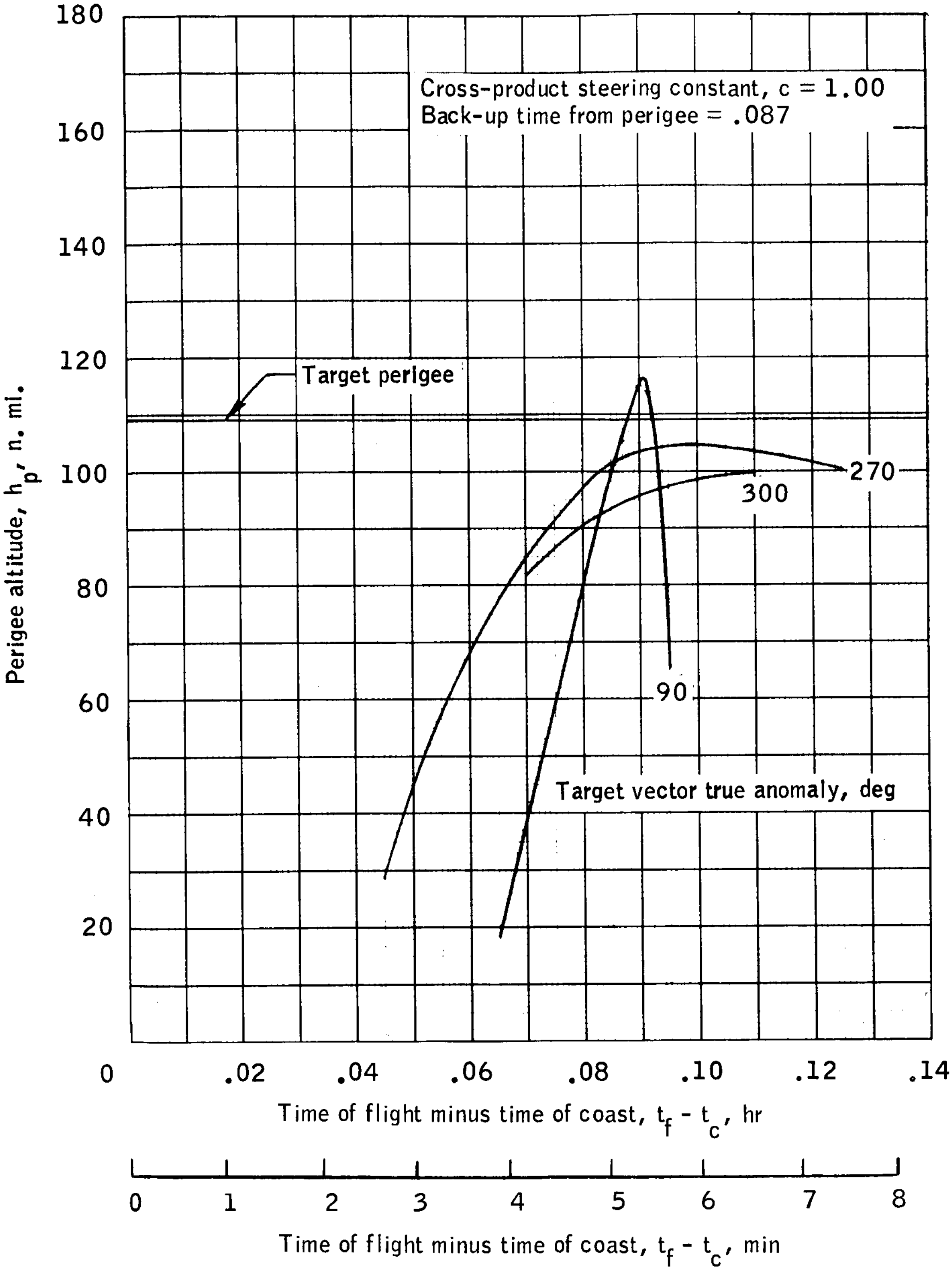

Figure 2.- Perigee altitude versus time of flight minus time of coast.

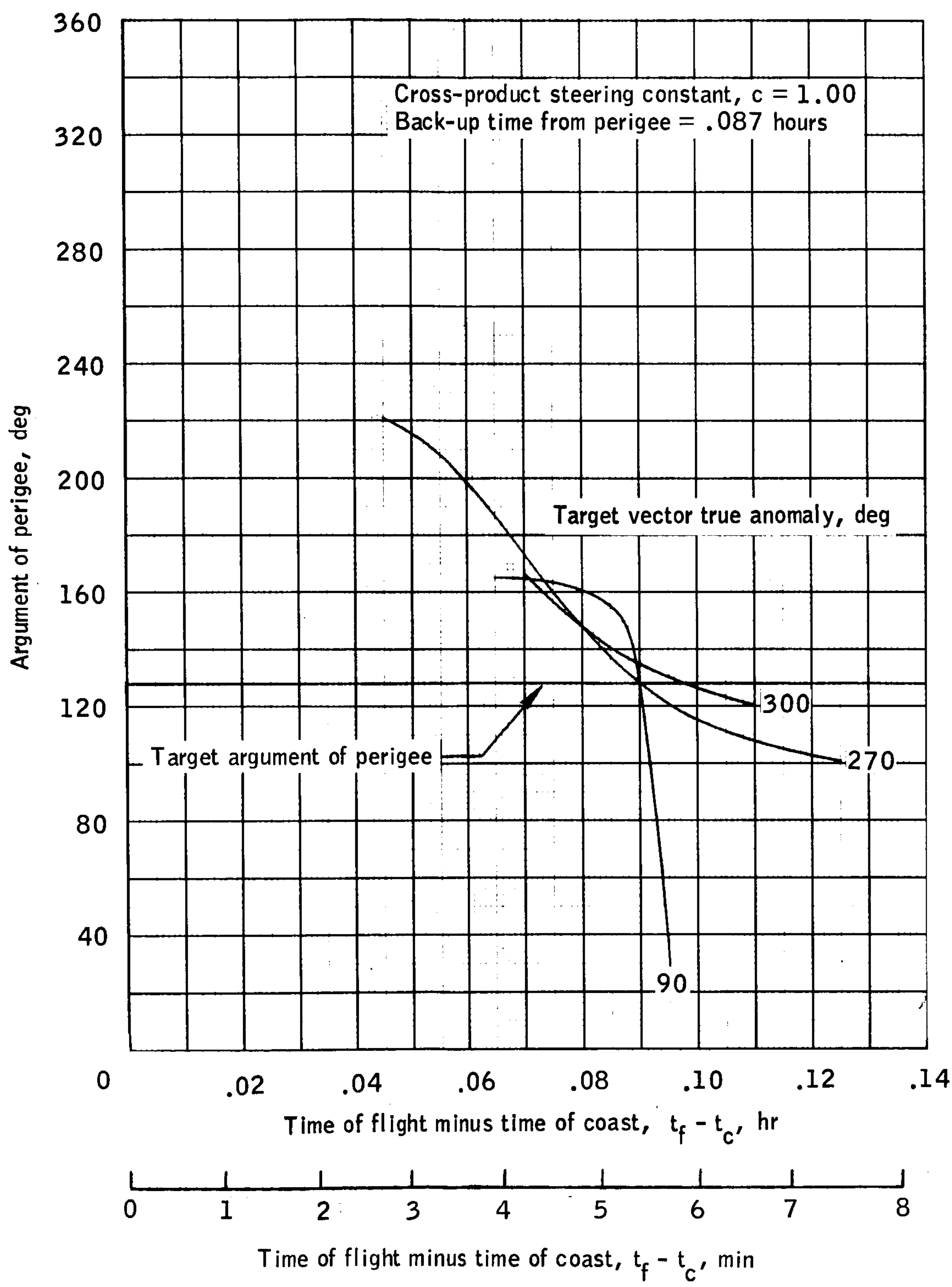

Figure 3.- Argument of perigee versus time of flight minus time of coast.

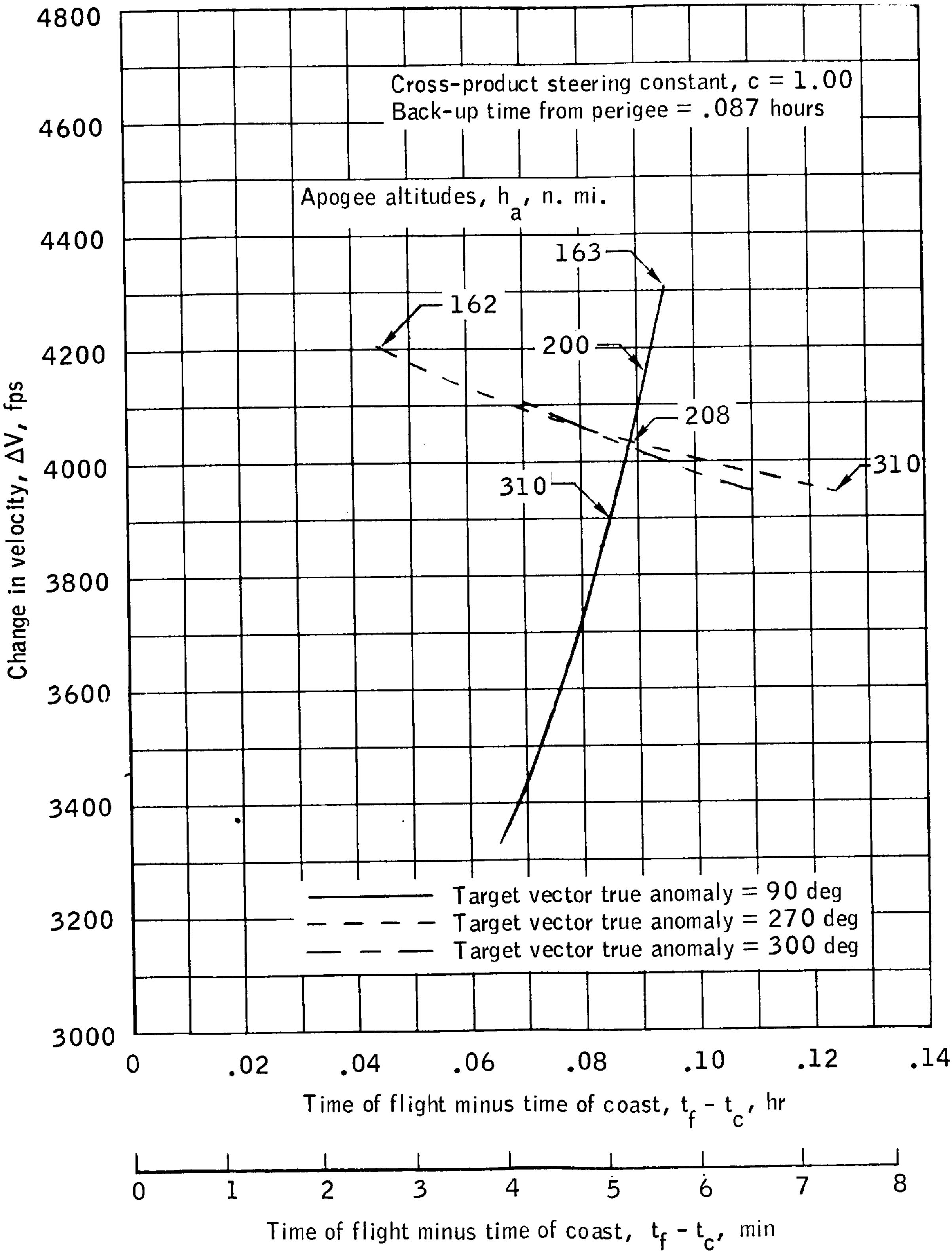

Figure 4.- Change in velocity versus time of flight minus time of coast.

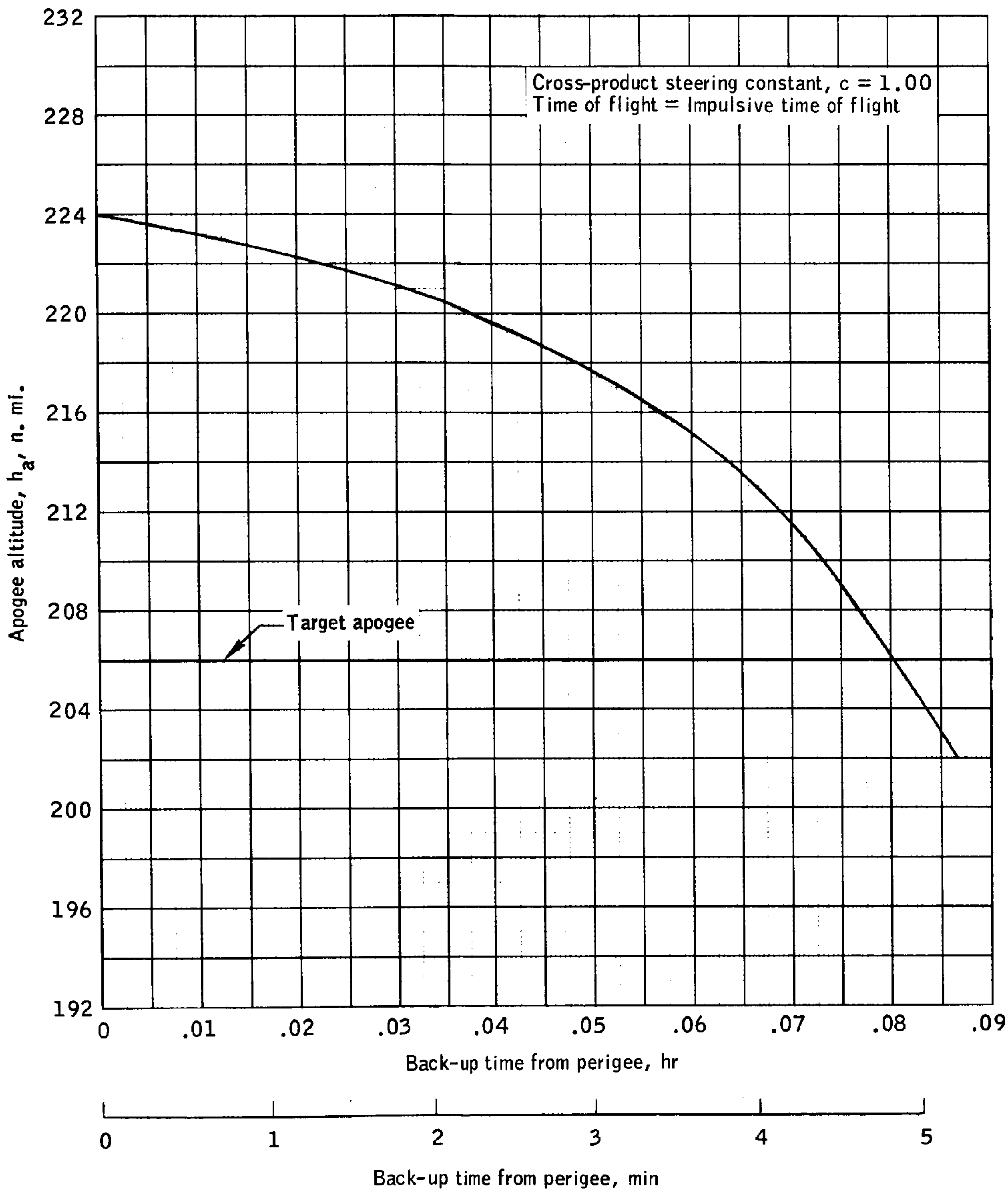

Figure 5. - Apogee altitude versus back-up time from perigee.

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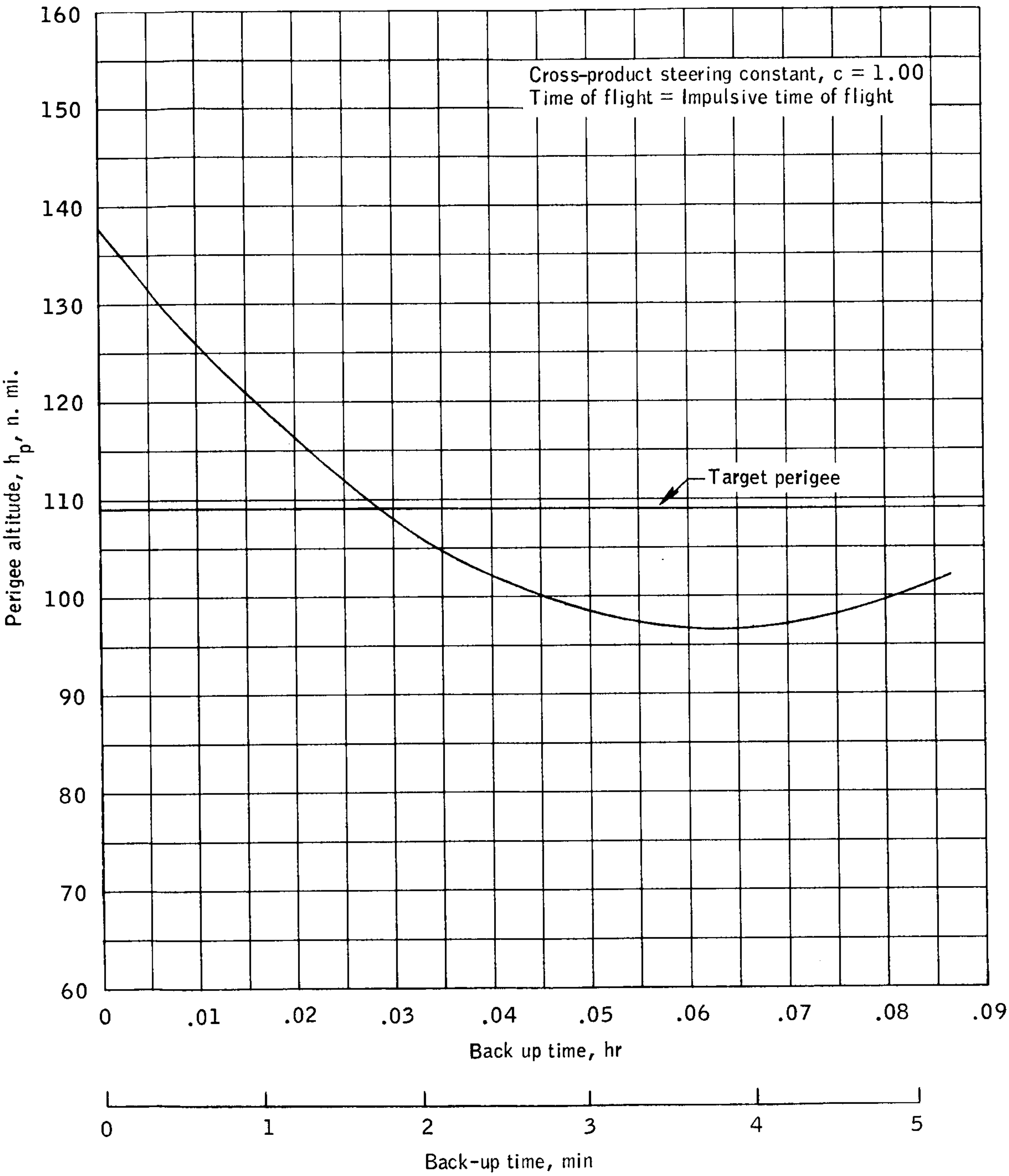

Figure 6.- Perigee altitude versus back up time from perigee.

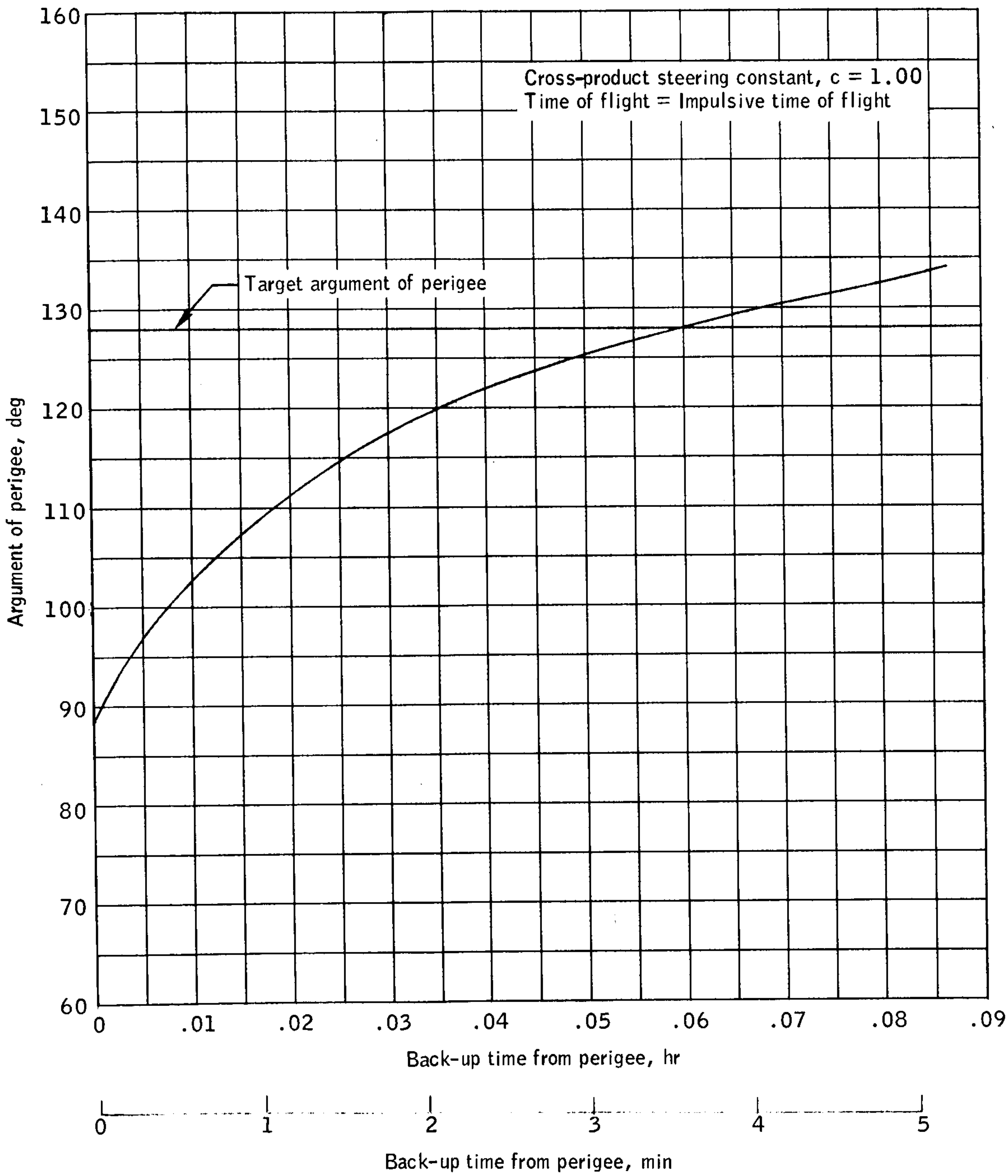

Figure 7.- Argument of perigee versus back-up time from perigee.

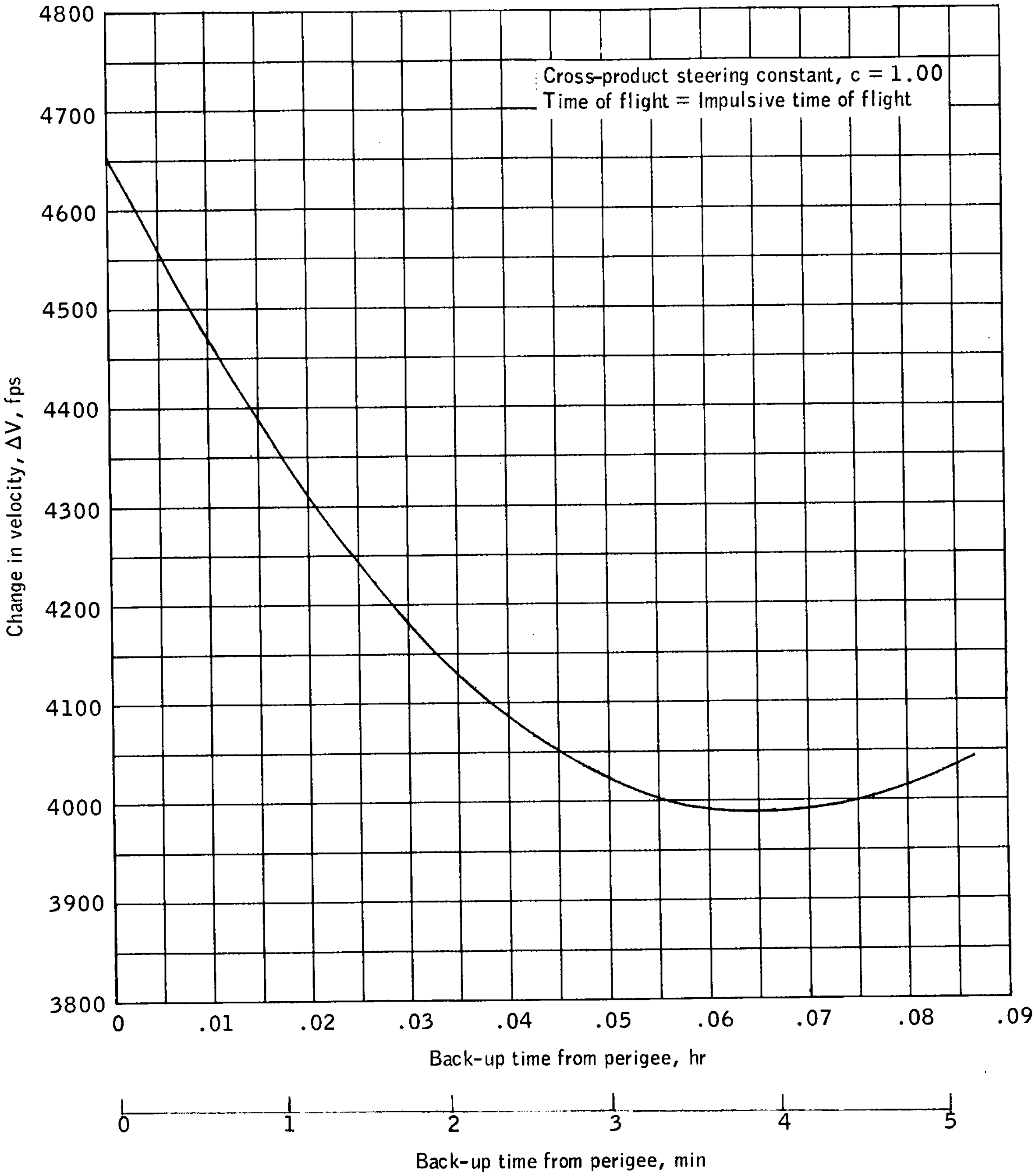

Figure 8.- Change in velocity versus back-up time from perigee.

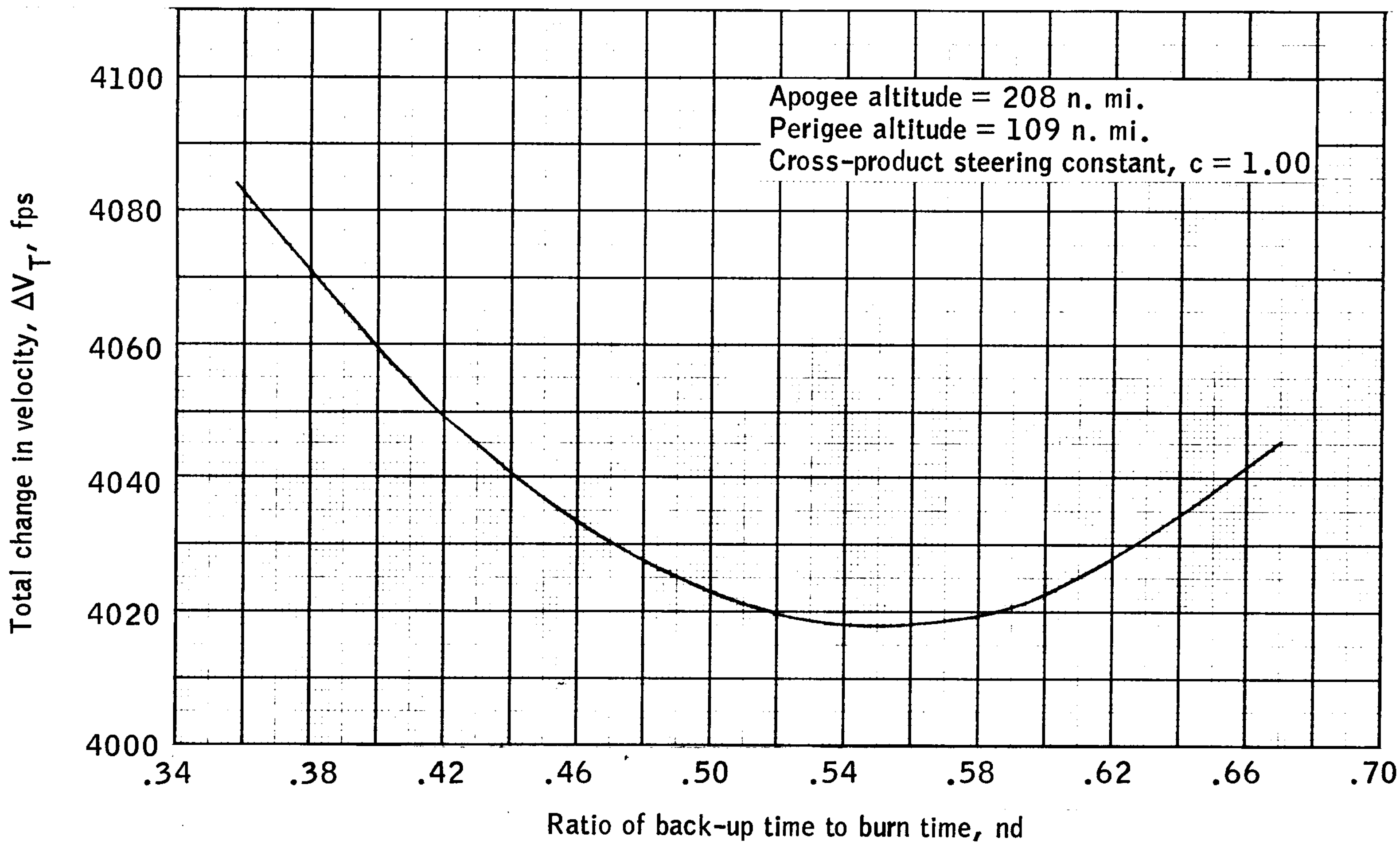

Figure 9.- Total change in velocity versus ratio of back-up time to burn time.

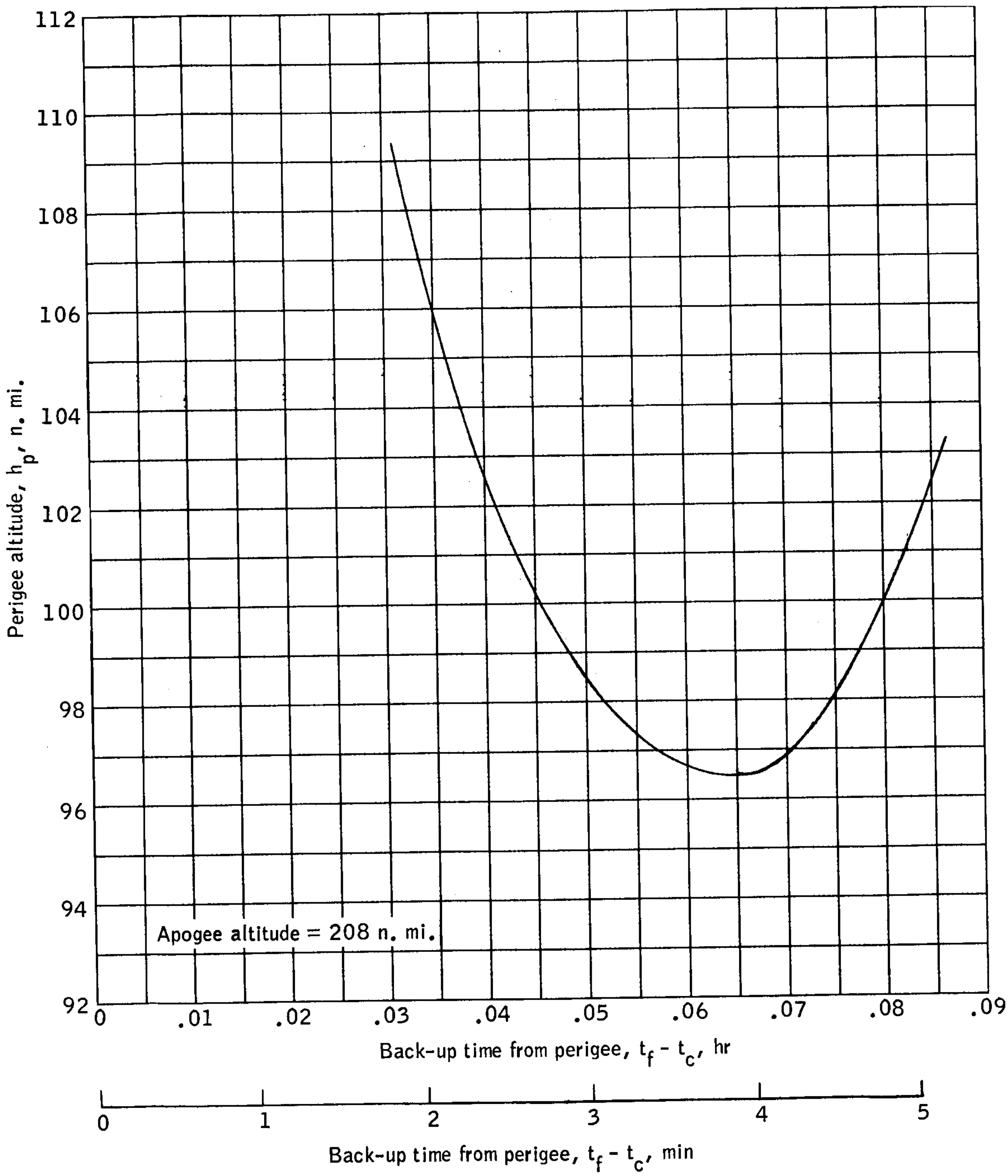

Figure 10.- Altitude of perigee versus back-up time from perigee.

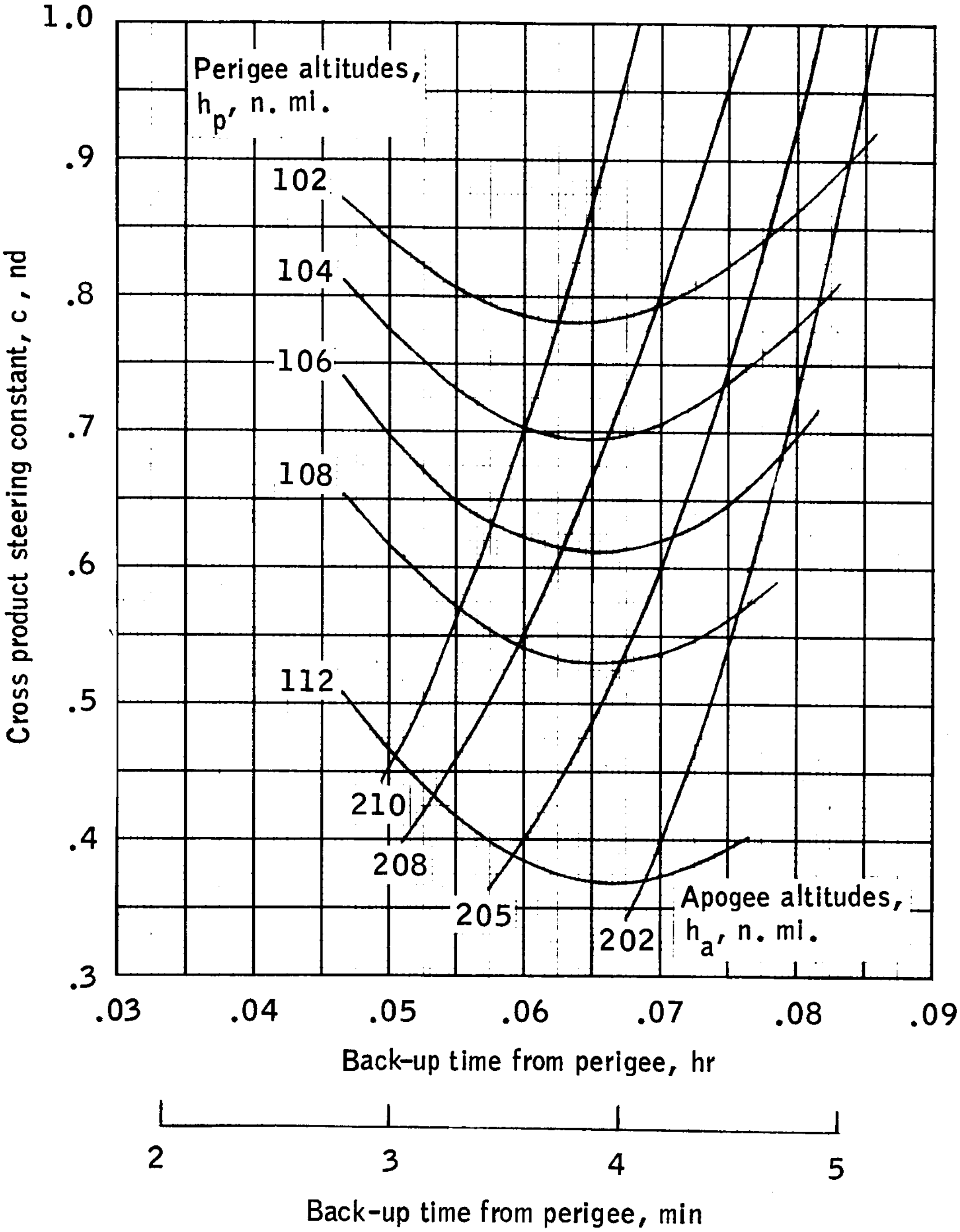

Figure 11.- Back-up time from perigee versus cross-product steering constant.

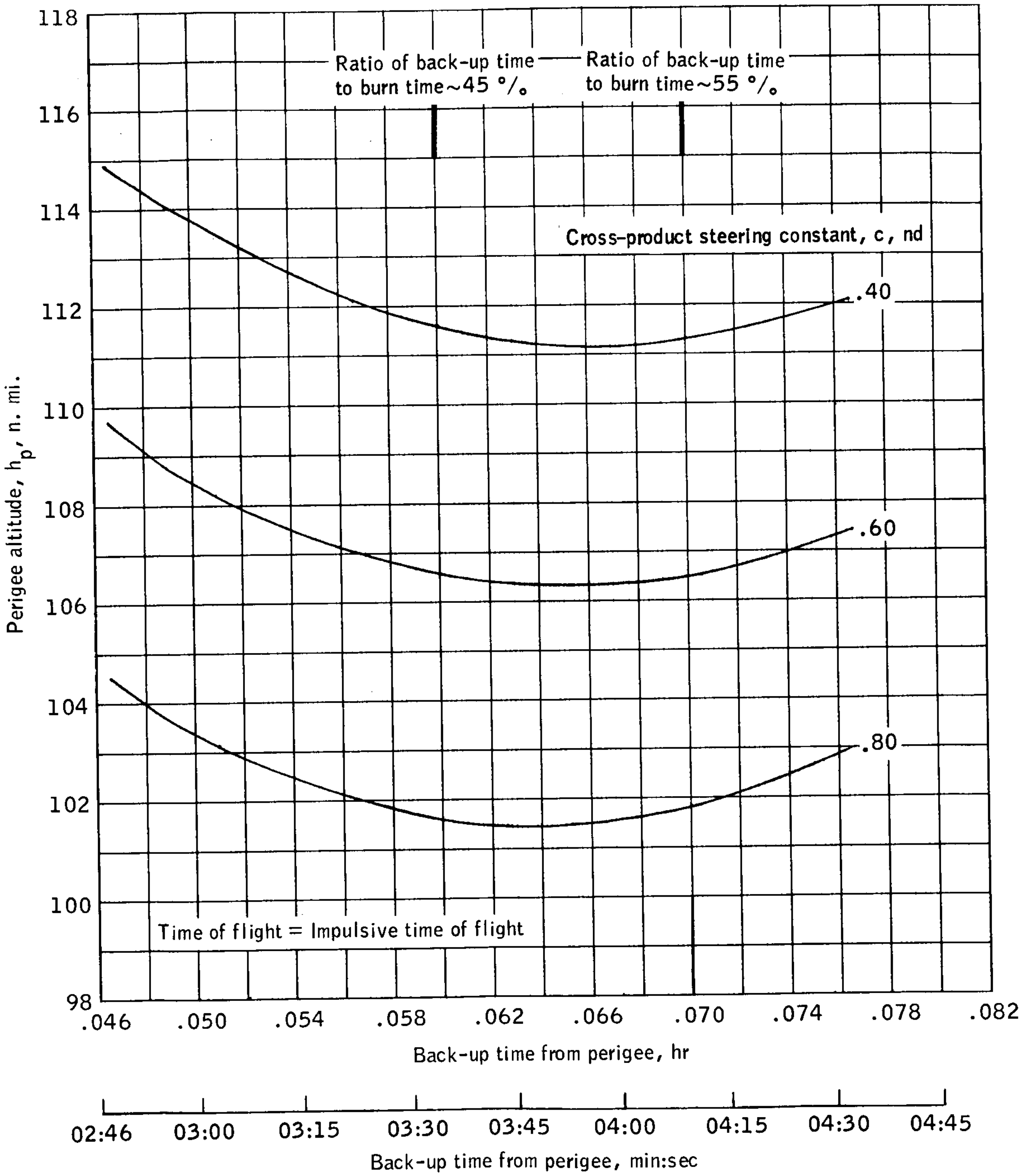

Figure 12.- Perigee altitude versus back-up time from perigee.

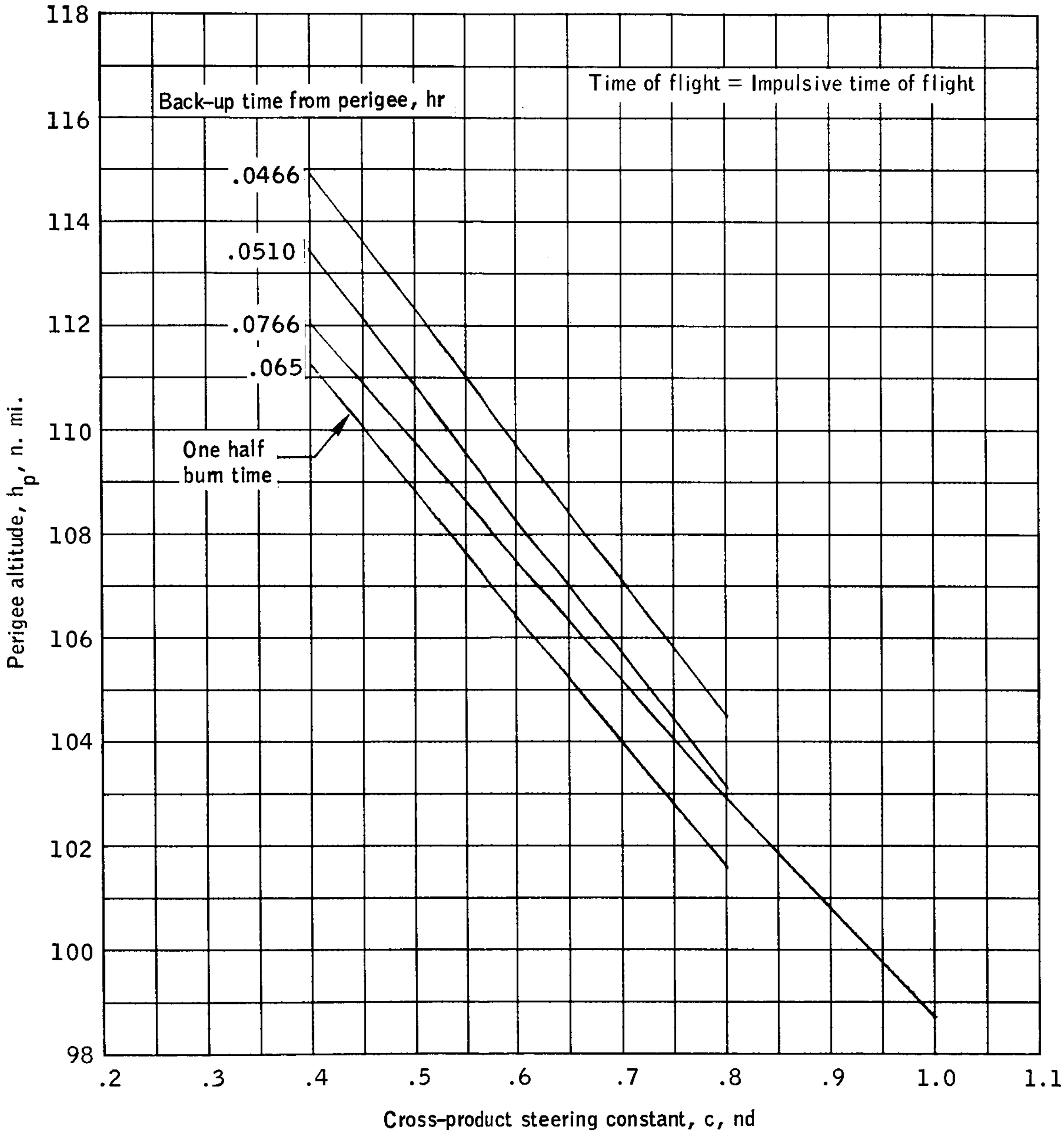

Figure 13.- Perigee altitude versus cross-product steering constant.

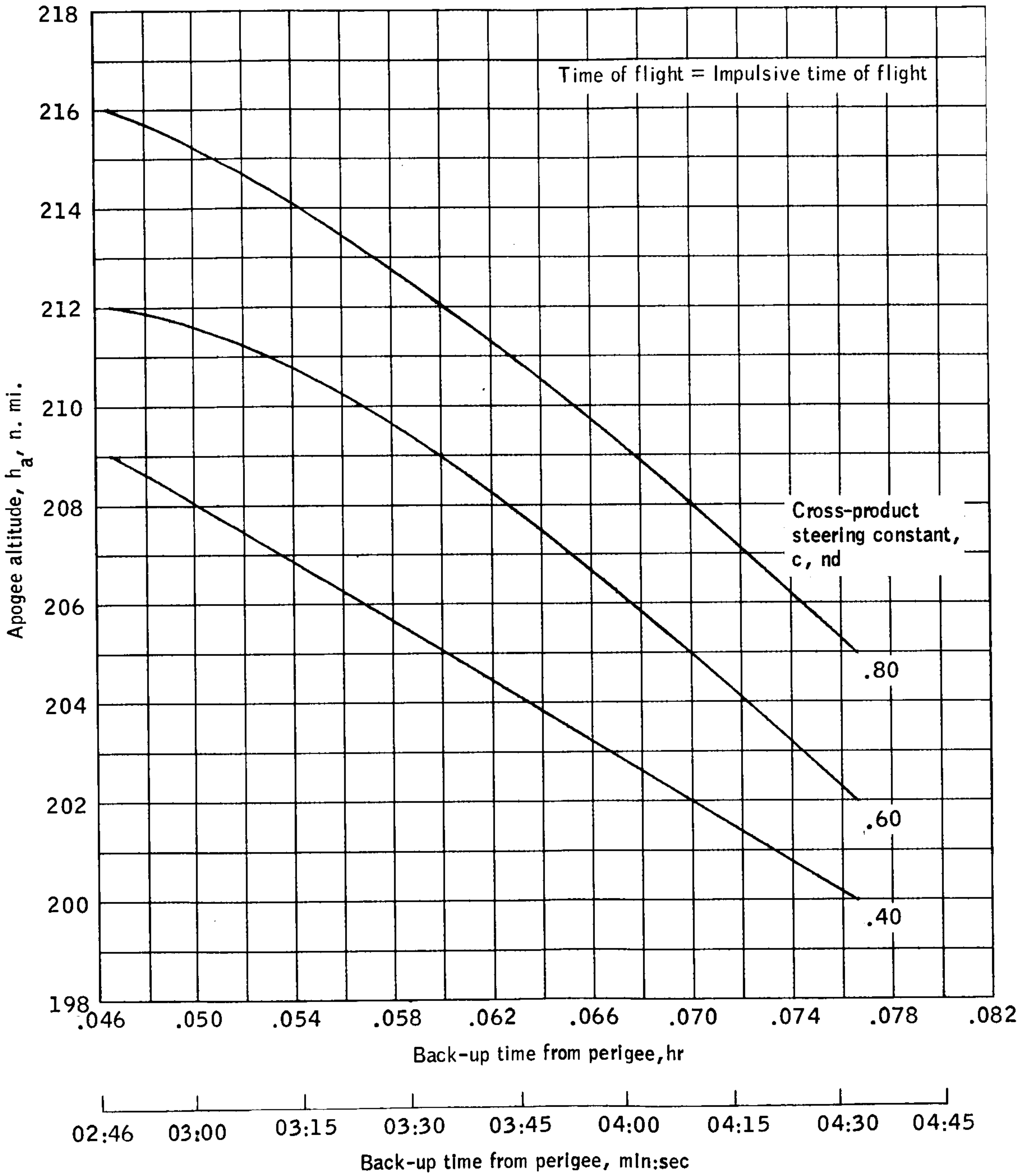

Figure 14.- Apogee altitude versus back-up time from perigee.

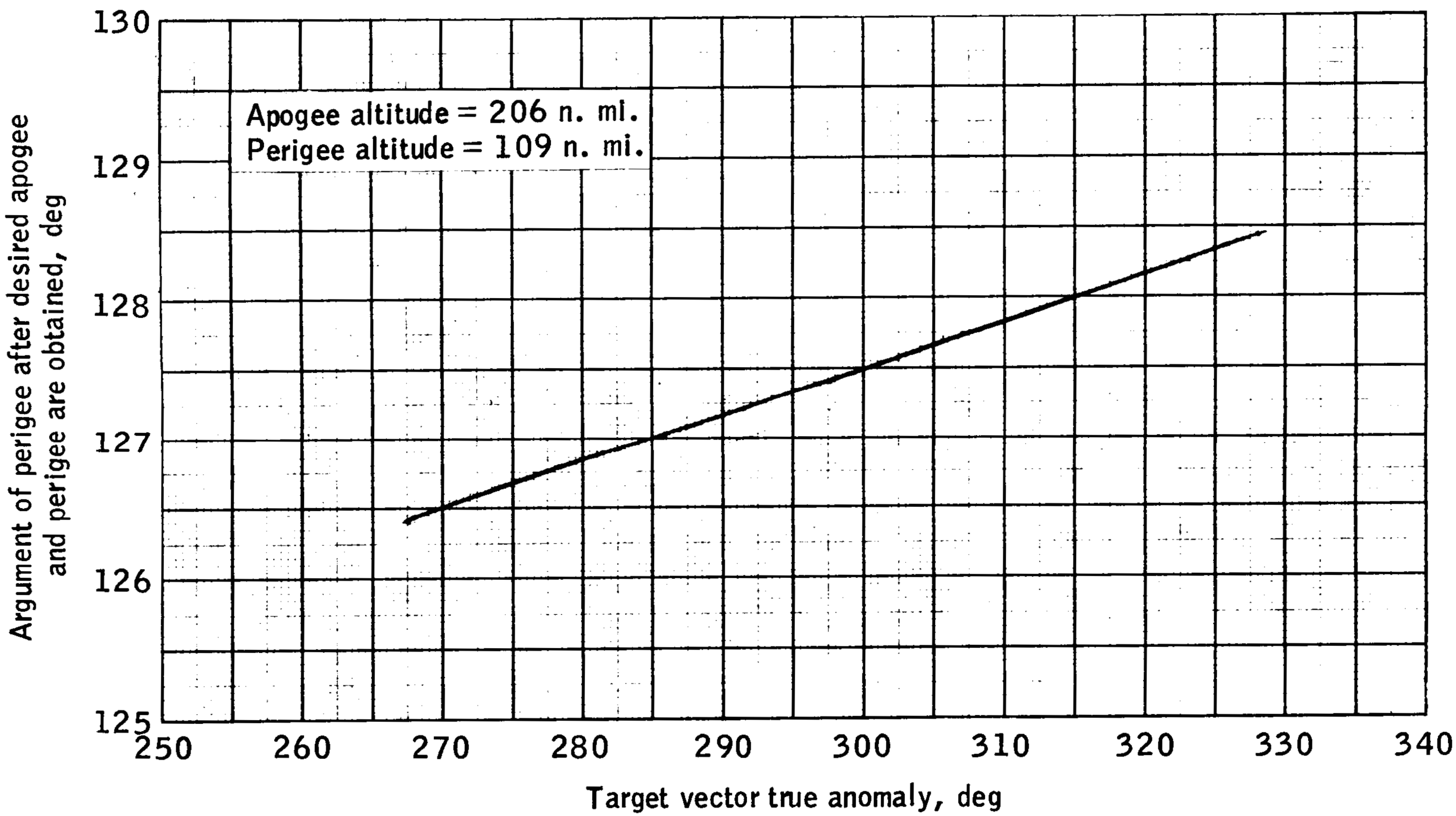

Figure 15.- Argument of perigee after desired apogee and perigee are obtained versus target vector true anomaly.

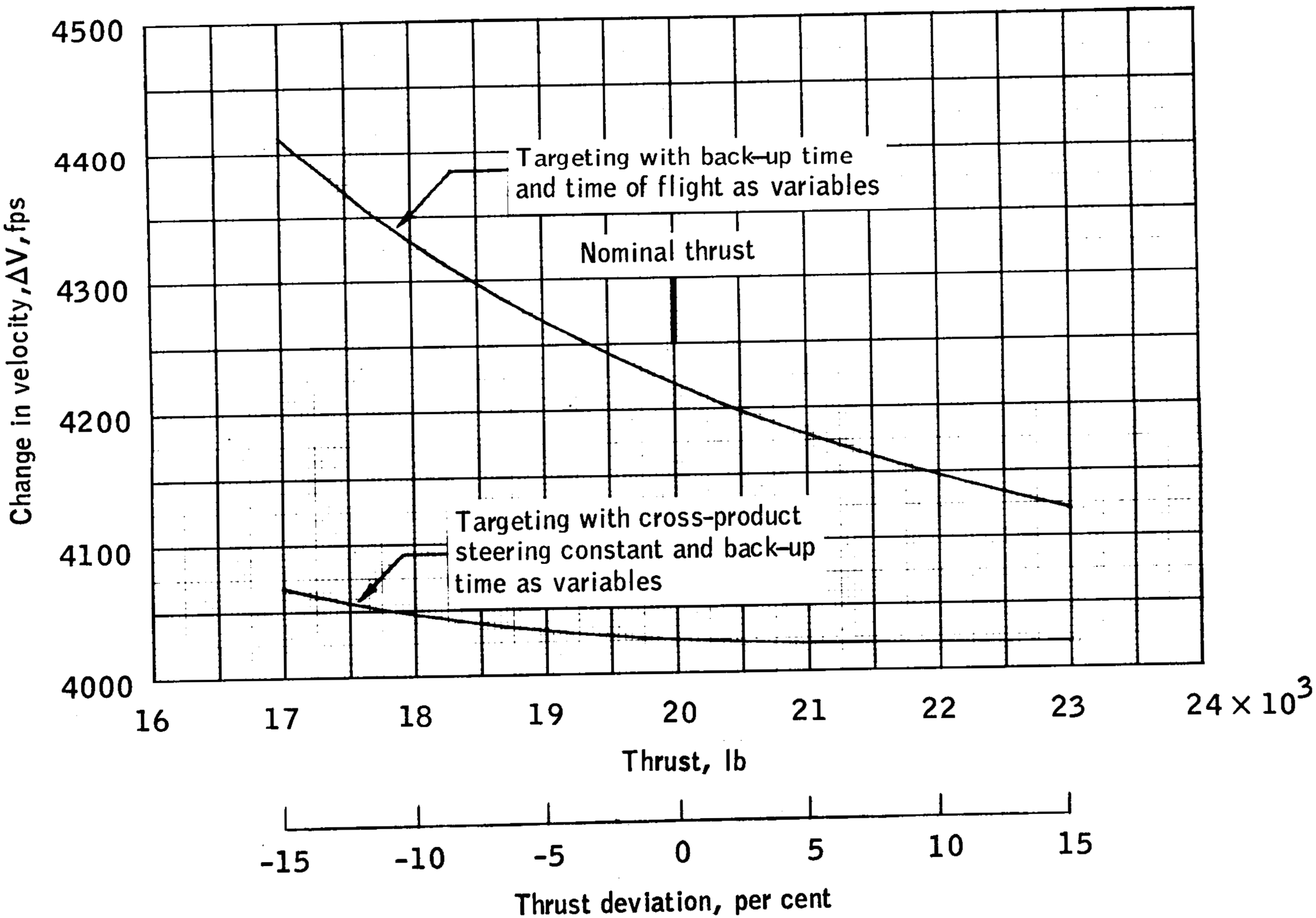

Figure 16.- Change in velocity versus thrust.

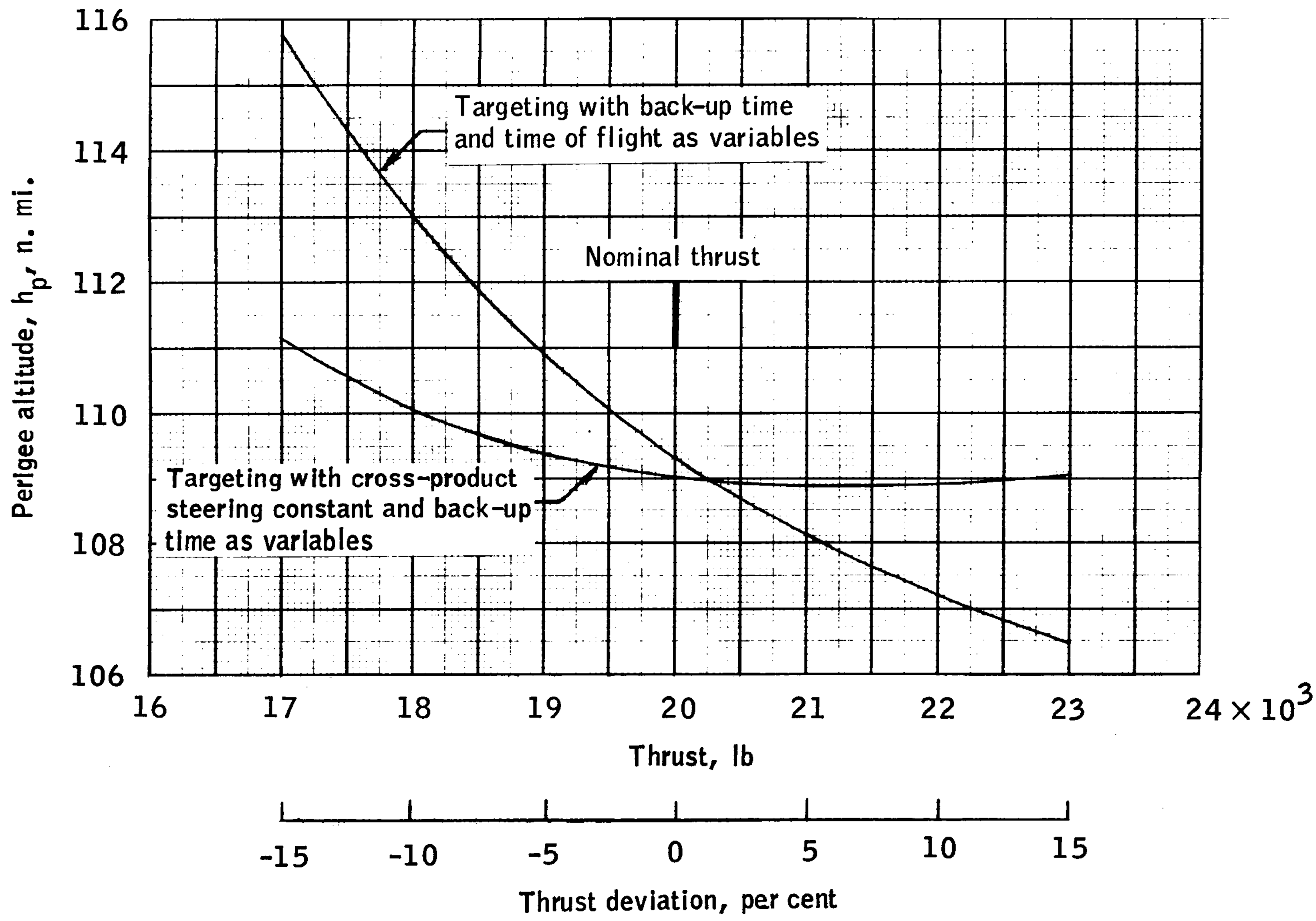

Figure 17.- Perigee altitude versus thrust.

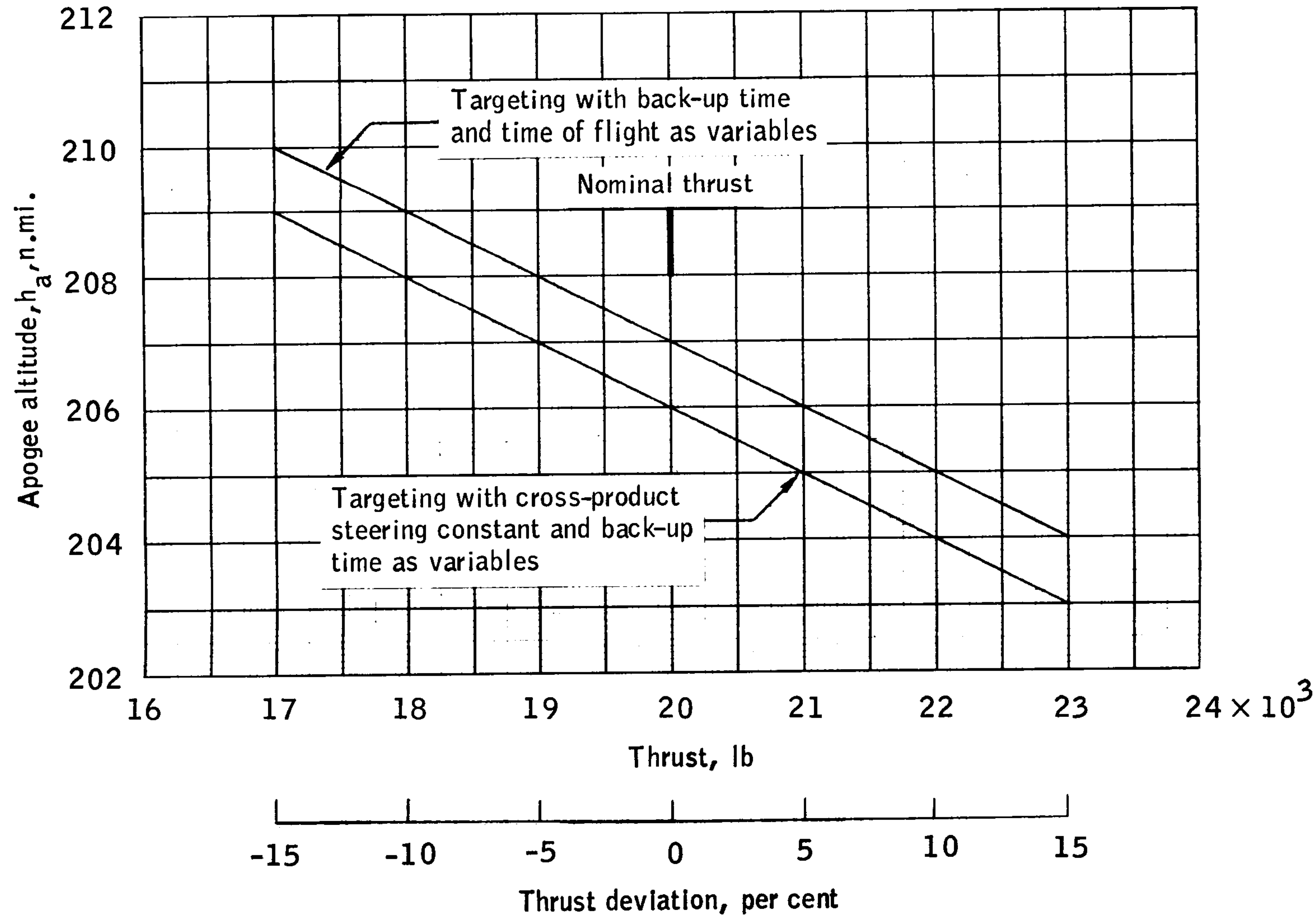

Figure 18.- Apogee altitude versus thrust.

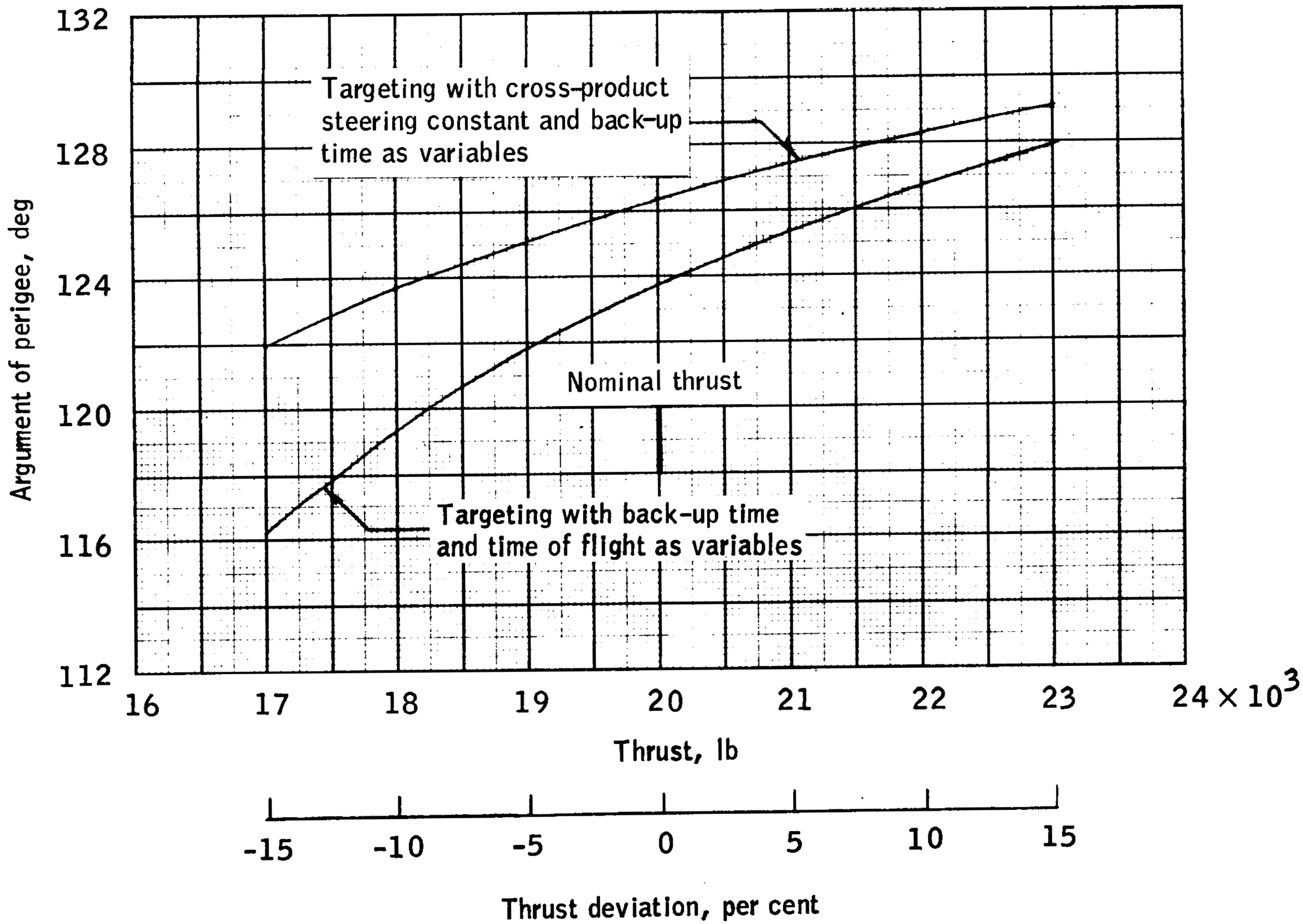

Figure 19.- Argument of perigee versus thrust.

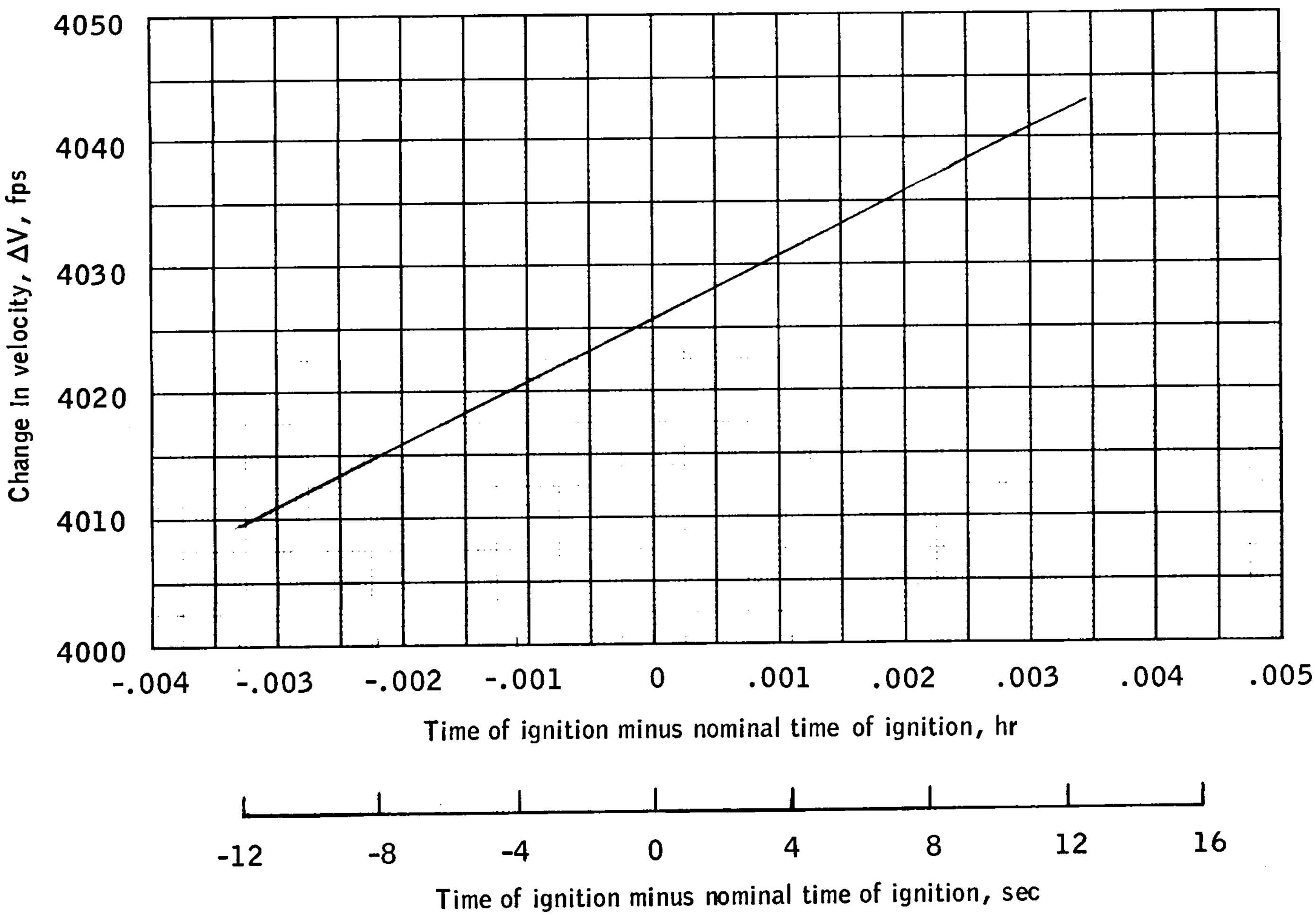

Figure 20.- Change in velocity versus time of ignition minus nominal time of ignition.

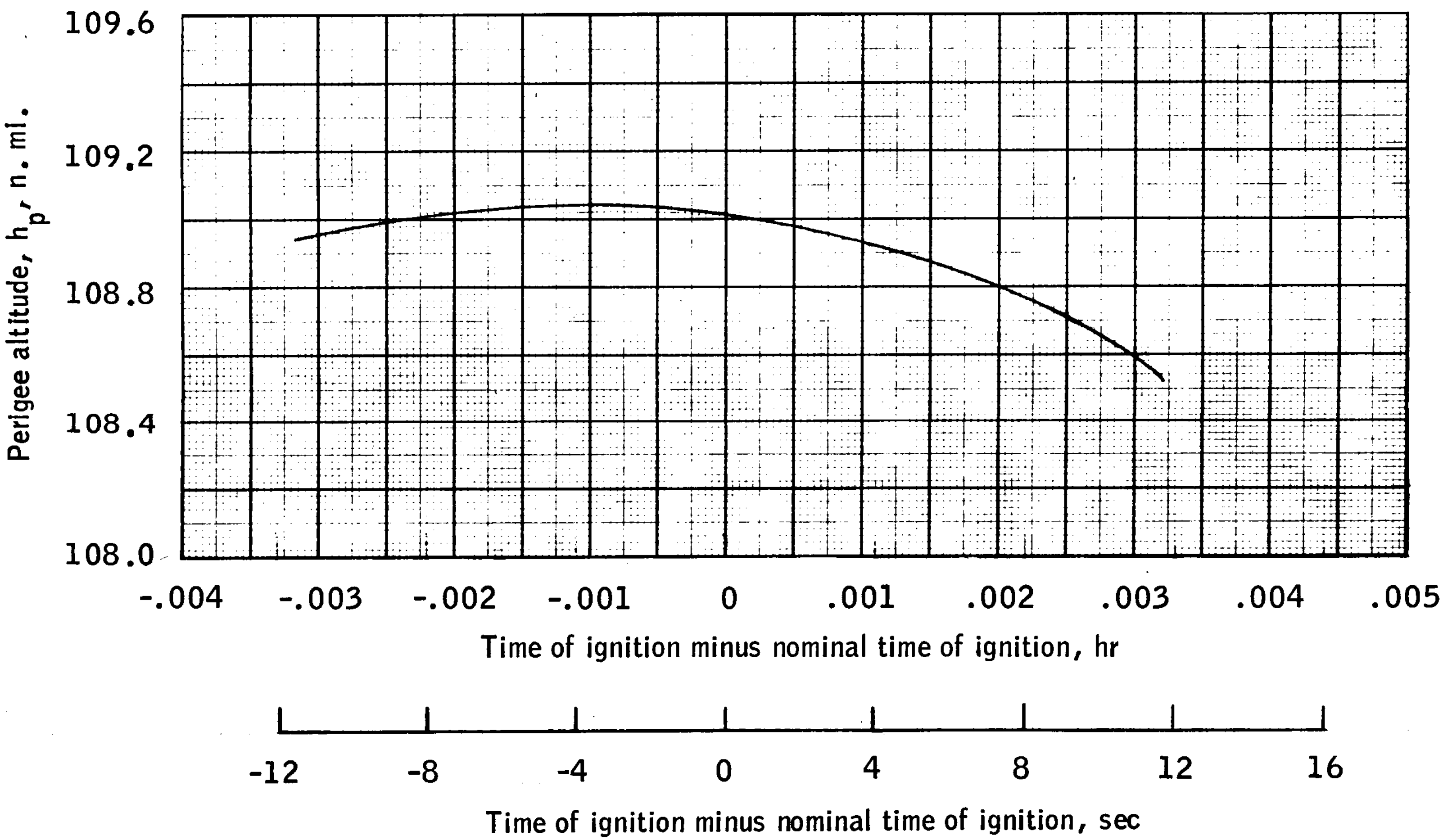

Figure 21.- Perigee altitude versus time of ignition minus nominal time of ignition.

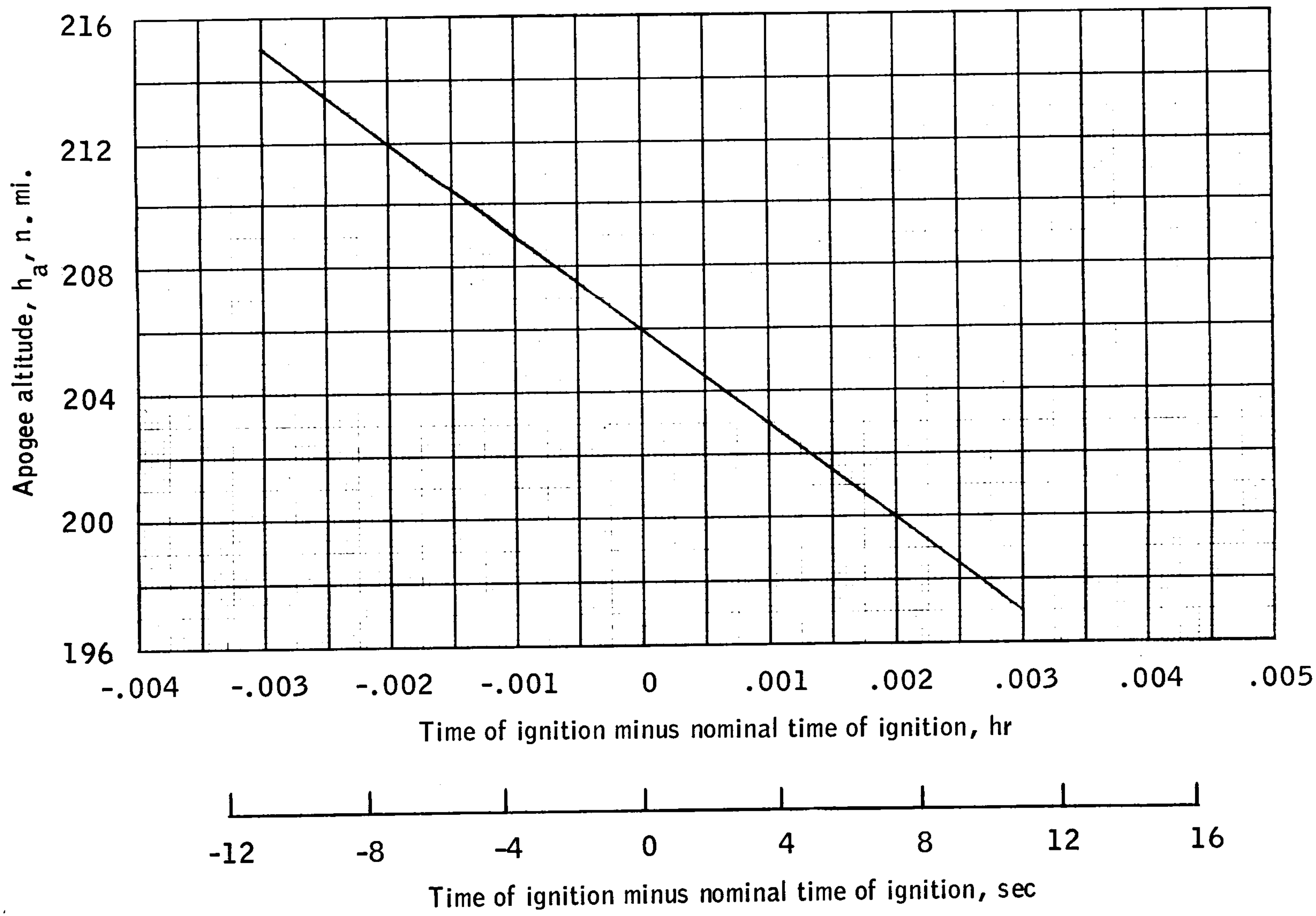

Figure 22.- Apogee altitude versus time of ignition minus nominal time of ignition.

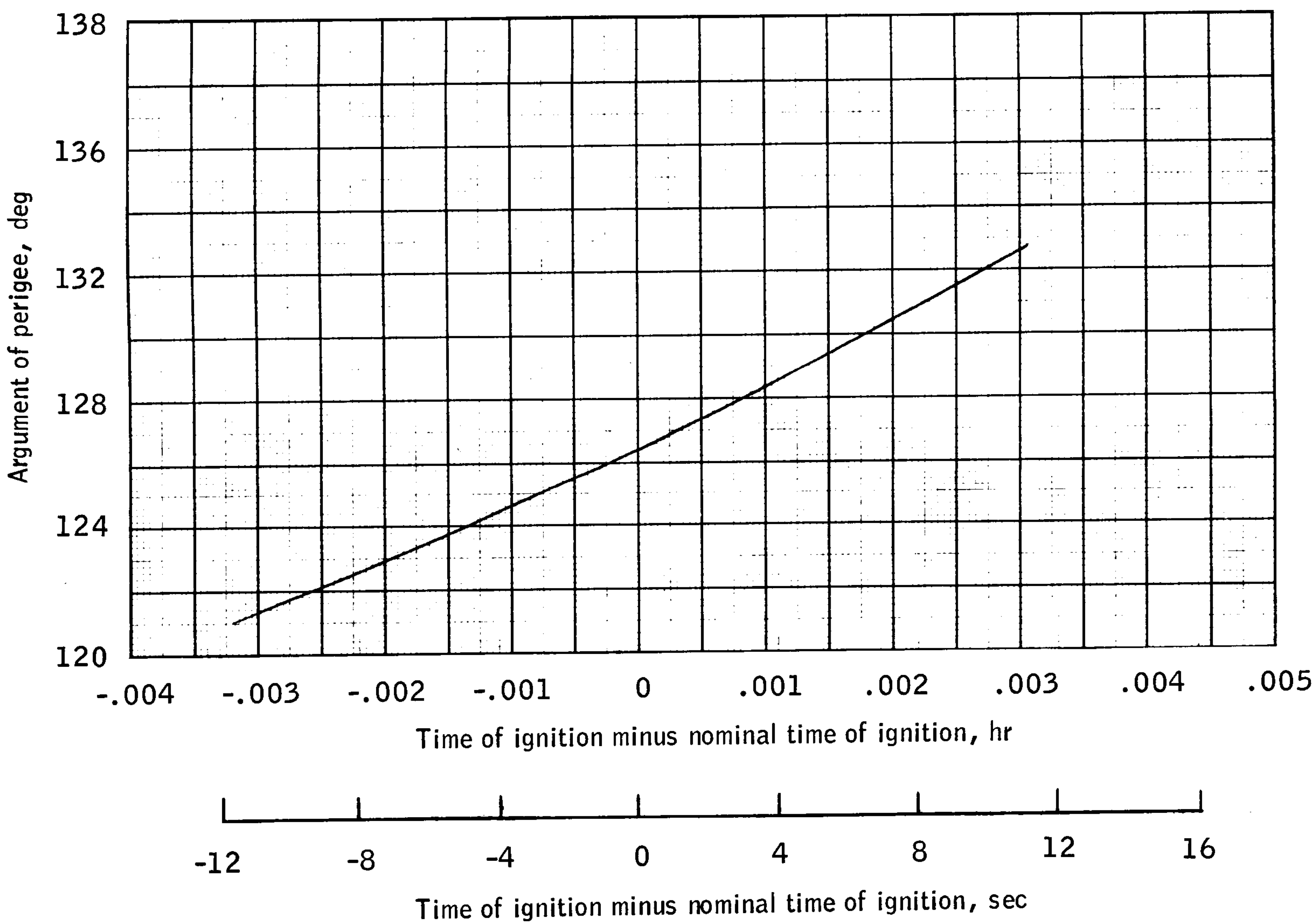

Figure 23.- Argument of perigee versus time of ignition minus nominal time of ignition.

APPENDIX

BURNS TARGETED BY THE GPMP

The tables in this appendix present the results of burns targeted by the GPMP part of the ARRS program. At perigee of the high ellipse, an impulsive maneuver was performed to raise apogee altitude. The orbital elements after this maneuver were propogated conically through 270°. This position served as the target vector. A burn time was calculated from the ideal rocket equation using the ΔV calculated for the impulsive maneuver; ignition time was then the time at perigee minus one-half the burn time. The time of flight was the time at the target vector minus the time of ignition. This is the method proposed for real-time updating.

TABLE AI.- BURN FROM A NOMINAL HIGH ELLIPSE

[Plane change = 0.0°; $c = 0.49$; $t_{bu} = 0.0637179$ hr; $t_{burn} = 0.12837650$ hr; $t_{bu}/t_{burn} = 0.49633$]

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps
Target	150	200	--	--
Orbital parameters preburn	148.9002	3950.1551	125.14543	--
Burn results measured at burnout	148.8102	204.0784	122.08997	4023.3779
Results at first perigee	151.164996	202.0425	120.31720	--

TABLE AII.- BURN FROM A HIGH ELLIPSE RESULTING FROM PREMATURE SHUTDOWN ON THE TLI SIMULATION BURN

[Plane change = 1.0°; $c = 0.49$; $t_{bu} = 0.0542024$ hr; $t_{burn} = 0.10890963$ hr; $t_{bu}/t_{burn} = 0.49768$]

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps
Target	150.0	200.1	124.3	--
Orbital parameters preburn	148.1686	2944.2246	123.03964	--
Burn results measured at burnout	148.0141	202.6453	123.35041	2207.1693
Results at first perigee	150.42944	201.0960	122.27173	--

TABLE AIII.- BURN FROM A HIGH ELLIPSE RESULTING FROM PREMATURE SHUTDOWN ON THE TLI SIMULATION BURN

[Plane change = 1.0°; $c = 0.49$; $t_{bu} = 0.0242501$ hr; $t_{burn} = 0.04873195$ hr; $t_{bu}/t_{burn} = 0.49762$]

	h_p, n. mi.	h_a, n. mi.	ω, deg	ΔV, fps
Target	150.0	200.0	117.3	--
Orbital parameters preburn	147.5007	1039.6278	115.86678	--
Burn results measured at burnout	147.4842	201.8438	117.06501	1351.2484
Results at first perigee	150.23274	201.2950	117.22142	--

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1. MIT Instrumentation Labortory: Guidance System Operations Plan AS-278; Vol. I, October 1966.

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